Lower Bounds on Formula Size of Boolean Functions using Hypergraph-Entropy

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Abstract
Korner [7] defined the notion of graph-entropy. He used it in [8] to simplify the proof of the Fredman-Komlos lower bound for the family size of perfect hash functions.

We use this information theoretic notion to obtain a general method for formula size lower bounds. This method can be applied to low-complexity functions for which the other known general methods ([11, 12, 3] and see also [17]) do not apply. Specifically the results are:

1. A new general lower bound on the formula size of quadratic Boolean functions.
2. As a corollary we get an $\Omega(n^2 logn)$ lower bound for the function that decides whether a graph of n vertices has a cycle of length four, and to the function that decides whether a graph has a vertex of degree at least two.
3. A simple proof of a result of Krichevskii, [10] , stating that the formula size for the threshold-2 Boolean function with n variables is at least $n log n$.
4. A simple proof of a lower bound first proved by Snir, [16], stating that a $WVW formula for n- variable threshold-k function, where all $^$ gates have fan in k, has the size of $\Omega(n{log n - log(k-1)\over {log k - log(k -1)}})=\Omega(nk log {n \over k})