

# Lower Bounds on Formula Size of Boolean Functions using Hypergraph-Entropy

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## Abstract

Korner [7] defined the notion of graph-entropy. He used it in [8] to simplify the proof of the Fredman-Komlos lower bound for the family size of perfect hash functions.

We use this information theoretic notion to obtain a general method for formula size lower bounds. This method can be applied to low-complexity functions for which the other known general methods ([11, 12, 3] and see also [17] ) do not apply. Specifically the results are:

1. A new general lower bound on the formula size of quadratic Boolean functions.
2. As a corollary we get an  $\Omega(n^2 \log n)$  lower bound for the function that decides whether a graph of  $n$  vertices has a cycle of length four, and to the function that decides whether a graph has a vertex of degree at least two.
3. A simple proof of a result of Krichevskii, [10] , stating that the formula size for the threshold-2 Boolean function with  $n$  variables is at least  $n \log n$ .
4. A simple proof of a lower bound first proved by Snir, [16], stating that a  $WVW$  formula for  $n$ - variable threshold- $k$  function, where all  $\wedge$  gates have fan in  $k$ , has the size of  $\Omega(n \frac{\log n - \log(k-1)}{\log k - \log(k-1)}) = \Omega(nk \log \frac{n}{k})$