

Neighborly Embedded Manifolds

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Abstract An embedding of an n -dimensional manifold M into R^d is called k -neighborly if, for every k points on the embedded manifold, there is a hyperplane H in R^d which supports the manifold precisely at these points.

Micha A. Perles (Problems presented in Oberwolfach conference on “Convexity”, 1982) asked: What is the smallest dimension $d(k, n)$ of the ambient space in which a k -neighborly n -dimensional manifold exists?

We prove that $d(k, n) \leq 2k(k - 1)n$. Related results and open problems are discussed.

Keywords Convex bodies · Polytopes · Neighborliness · Cyclic polytopes · Continuous hashing

1 Introduction

An embedding of an n -dimensional (connected) manifold M into R^d is called k -neighborly if, for every k points on the embedded manifold, there is a hyperplane H in R^d which supports the manifold precisely at these points. Namely, H contains these k points and all other points of the embedded manifold are in the same open half space determined by H .

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