

# Neighborly Embedded Manifolds

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**Abstract** An embedding of an  $n$ -dimensional manifold  $M$  into  $R^d$  is called  $k$ -neighborly if, for every  $k$  points on the embedded manifold, there is a hyperplane  $H$  in  $R^d$  which supports the manifold precisely at these points.

Micha A. Perles (Problems presented in Oberwolfach conference on “Convexity”, 1982) asked: What is the smallest dimension  $d(k, n)$  of the ambient space in which a  $k$ -neighborly  $n$ -dimensional manifold exists?

We prove that  $d(k, n) \leq 2k(k - 1)n$ . Related results and open problems are discussed.

**Keywords** Convex bodies · Polytopes · Neighborliness · Cyclic polytopes · Continuous hashing

## 1 Introduction

An embedding of an  $n$ -dimensional (connected) manifold  $M$  into  $R^d$  is called  $k$ -neighborly if, for every  $k$  points on the embedded manifold, there is a hyperplane  $H$  in  $R^d$  which supports the manifold precisely at these points. Namely,  $H$  contains these  $k$  points and all other points of the embedded manifold are in the same open half space determined by  $H$ .

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