Robust sublinear expanders

Matija Bucić

Institute for Advanced Study and Princeton University

Expander graphs

- Expander graphs: "robustly well-connected graphs"

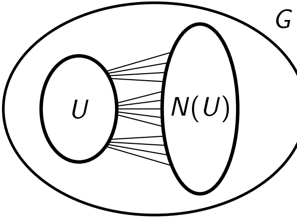
Expander graphs

- Expander graphs: "robustly well-connected graphs"

Definition (Expansion)

An n-vertex graph G is a λ-expander if for all $U \subseteq V(G)$ s.t. $|U| \leq \frac{n}{2\lambda}$ we have:

$$|N(U)| > \lambda |U|.$$

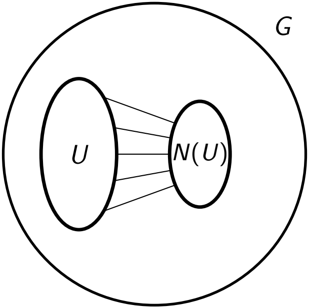


Sublinear expansion

Definition (Sublinear expansion)

An n-vertex graph G is a sublinear expander if $\forall U \subseteq V(G)$ *s.t.* $|U| \leq \frac{n}{2}$ *we have:*

$$|N(U)| > \frac{1}{\log^2 n} \cdot |U|$$

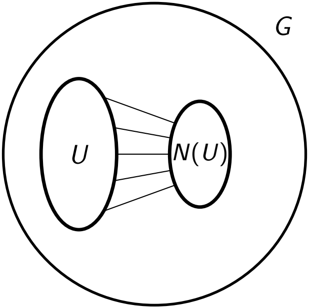


Sublinear expansion

Definition (Sublinear expansion)

An n-vertex graph G is a sublinear expander if $\forall U \subseteq V(G)$ s.t. $|U| \leq \frac{n}{2}$ we have:

$$|N(U)| > \frac{1}{\log^2 n} \cdot |U|$$



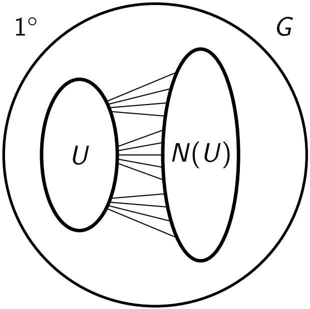
- Komlós and Szemerédi: Can find a sublinear expander of almost the same average degree in any graph.

Robust sublinear expansion

Definition (Robust sublinear expansion)

An n-vertex graph G is a robust sublinear expander if $\forall U \subseteq V(G): |U| \le \frac{n}{2}$

$$1^\circ: \quad |N(U)| > \log^2 n \cdot |U| \qquad \textit{or}$$



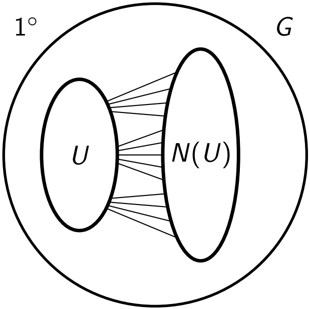
Robust sublinear expansion

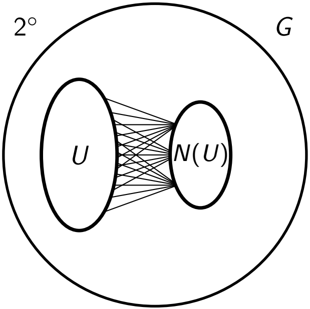
Definition (Robust sublinear expansion)

An n-vertex graph G is a robust sublinear expander if $\forall U \subseteq V(G): |U| \le \frac{n}{2}$

$$1^\circ: \quad |N(U)| > \log^2 n \cdot |U| \qquad \textit{or}$$

$$2^\circ: \quad N(U) \textit{ has } > \frac{|U|}{\log^2 n} \textit{ vtcs with } \ge \log^4 n \textit{ neighbours in } U.$$





Graph decomposition problems

- General graph decomposition question:

Graph decomposition problems

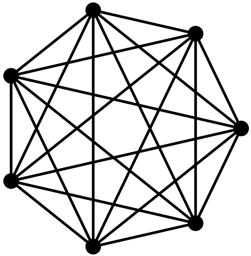
- General graph decomposition question:

  Can we decompose a graph into few graphs with some "nice" properties?

Graph decomposition problems

- General graph decomposition question:

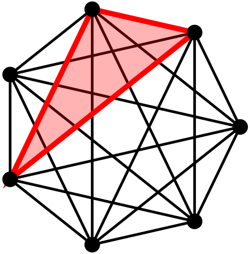
  Can we decompose a graph into few graphs with some "nice" properties?



Graph decomposition problems

- General graph decomposition question:

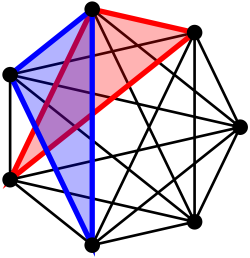
  Can we decompose a graph into few graphs with some "nice" properties?



Graph decomposition problems

- General graph decomposition question:

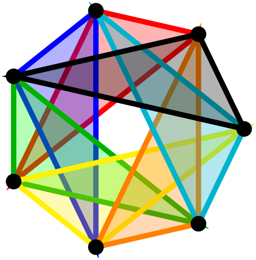
  Can we decompose a graph into few graphs with some "nice" properties?



Graph decomposition problems

- General graph decomposition question:

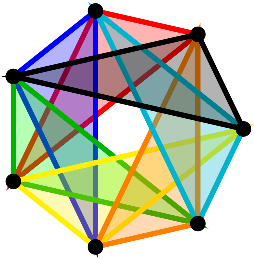
  Can we decompose a graph into few graphs with some “nice” properties?



Graph decomposition problems

- General graph decomposition question:

  Can we decompose a graph into few graphs with some "nice" properties?

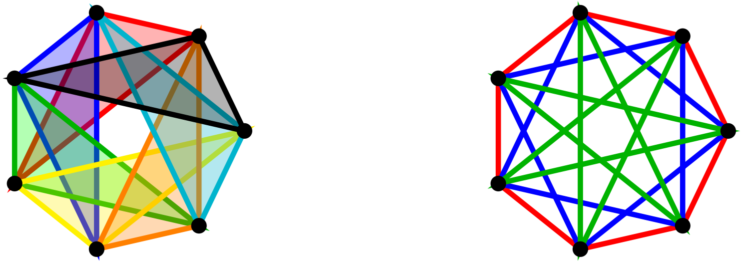


- Walecki 1883: K_{2n+1} can be decomposed into n cycles.

Graph decomposition problems

- General graph decomposition question:

  Can we decompose a graph into few graphs with some "nice" properties?

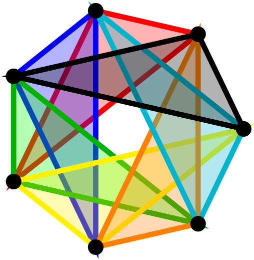


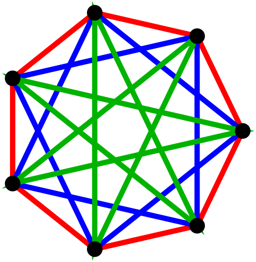
- Walecki 1883: K_{2n+1} can be decomposed into n cycles.

Graph decomposition problems

- General graph decomposition question:

  Can we decompose a graph into few graphs with some "nice" properties?



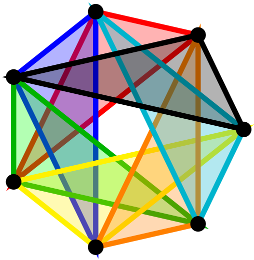


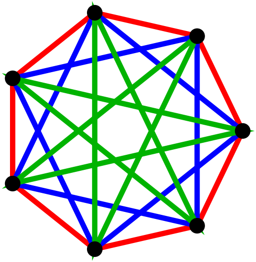
- Walecki 1883: K_{2n+1} can be decomposed into n cycles.
- Can we decompose any graph into cycles?

Graph decomposition problems

- General graph decomposition question:

  Can we decompose a graph into few graphs with some "nice" properties?





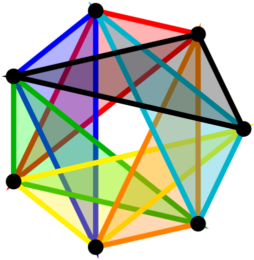
- Walecki 1883: K_{2n+1} can be decomposed into n cycles.

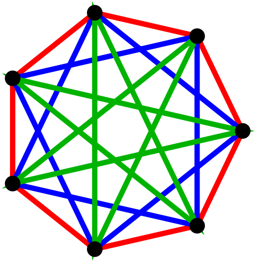
- Can we decompose any graph into cycles? No if $\exists$ an odd degree vertex.

Graph decomposition problems

- General graph decomposition question:

  Can we decompose a graph into few graphs with some "nice" properties?

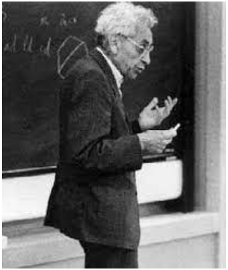




- Walecki 1883: K_{2n+1} can be decomposed into n cycles.
- Can we decompose any graph into cycles? No if $\exists$ an odd degree vertex.
- Veblen 1912: Any graph with all degrees even decomposes into cycles.

Conjecture (Erdős-Gallai 1960s)

Every n-vertex graph can be decomposed into $O(n)$ cycles and edges.



Conjecture (Erdős-Gallai 1960s)

Every n-vertex graph can be decomposed into $O(n)$ cycles and edges.

- Tight if true.

Erdős-Gallai Conjecture

Conjecture (Erdős-Gallai 1960s)

Every n-vertex graph can be decomposed into $O(n)$ cycles and edges.

- Tight if true.
  - Erdős 1983: one needs at least $(3/2 - o(1))n$ cycles and edges.

Erdős-Gallai Conjecture: related work

Conjecture (Erdős-Gallai 1960s)

Every n-vertex graph can be decomposed into $O(n)$ cycles and edges.

- Tight if true.
  - Erdős 1983: one needs at least $(3/2 - o(1))n$ cycles and edges.
- Lovász 1968: True for paths in place of cycles

What do we know?

Conjecture (Erdős-Gallai 1960s)

Every n-vertex graph can be decomposed into $O(n)$ cycles and edges.

What do we know?

Conjecture (Erdős-Gallai 1960s)

Every n-vertex graph can be decomposed into $O(n)$ cycles and edges.

- Proved for graphs with linear minimum degree.

What do we know?

Conjecture (Erdős-Gallai 1960s)

Every n-vertex graph can be decomposed into $O(n)$ cycles and edges.

- Proved for graphs with linear minimum degree.
- Proved for random graphs.

What do we know?

Conjecture (Erdős-Gallai 1960s)

Every n-vertex graph can be decomposed into $O(n)$ cycles and edges.

- Proved for graphs with linear minimum degree.
- Proved for random graphs.
- Folklore: $O(n \log n)$ cycles and edges always suffice.

What do we know?

Conjecture (Erdős-Gallai 1960s)

Every n-vertex graph can be decomposed into $O(n)$ cycles and edges.

- Proved for graphs with linear minimum degree.
- Proved for random graphs.
- Folklore: $O(n \log n)$ cycles and edges always suffice.
- Conlon, Fox and Sudakov: $O(n \log \log n)$ cycles and edges always suffice.

What do we know?

Conjecture (Erdős-Gallai 1960s)

Every n-vertex graph can be decomposed into $O(n)$ cycles and edges.

- Proved for graphs with linear minimum degree.
- Proved for random graphs.
- Folklore: $O(n \log n)$ cycles and edges always suffice.
- Conlon, Fox and Sudakov: $O(n \log \log n)$ cycles and edges always suffice.

Theorem (B., Montgomery 2022+)

Any n-vertex graph can be decomposed into $O(n \log^{\star} n)$ cycles and edges.

Erdős unit distance problem

Question (Erdős, 1946)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^2, \|.\|_2)$*?*

Erdős unit distance problem

Question (Erdős, 1946)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^2, \|.\|_2)$*?*

Let $U_{\|.\|_2}(n)$ denote the answer.

Erdős unit distance problem

Question (Erdős, 1946)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^2, \|.\|_2)$*?*

Let $U_{\|.\|_2}(n)$ denote the answer.

$$U_{||.||_2}(n) \qquad \underbrace{\leq O(n^{4/3})}_{\text{Spencer, Szemerédi, Trotter '84}}$$

Erdős unit distance problem

Question (Erdős, 1946)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^2, \|.\|_2)$?

Let $U_{\|.\|_2}(n)$ denote the answer.

$$\underbrace{\Omega\left(n^{1+\frac{1}{\log\log n}}\right) \leq}_{\text{Erdős '46}} \qquad U_{||.||_2}(n) \qquad \underbrace{\leq O(n^{4/3})}_{\text{Spencer, Szemerédi, Trotter '84}}$$

Erdős unit distance problem

Question (Erdős, 1946)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^2, \|.\|_2)$?

Let $U_{\|.\|_2}(n)$ denote the answer.

In 2D: $\underbrace{\Omega\left(n^{1+\frac{1}{\log\log n}}\right) \leq}_{\text{Erdős '46}} \quad U_{||.||_2}(n) \quad \underbrace{\leq O(n^{4/3})}_{\text{Spencer, Szemerédi, Trotter '84}}$

Erdős unit distance problem

Question (Erdős, 1946)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^d, \|.\|_2)$*?*

Let $U_{\|.\|_2}(n)$ denote the answer.

In 2D: $\underbrace{\Omega\left(n^{1+\frac{1}{\log\log n}}\right) \leq}_{\text{Erdős '46}} \quad U_{\|.\|_2}(n) \quad \underbrace{\leq O(n^{4/3})}_{\text{Spencer, Szemerédi, Trotter '84}}$

Erdős unit distance problem

Question (Erdős, 1946)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^d, \|.\|_2)$?

Let $U_{\|.\|_2}(n)$ denote the answer.

In 2D: $\underbrace{\Omega\left(n^{1+\frac{1}{\log\log n}}\right) \leq}_{\text{Erdős '46}} \quad U_{\|.\|_2}(n) \quad \underbrace{\leq O(n^{4/3})}_{\text{Spencer, Szemerédi, Trotter '84}}$

In 3D: $U_{\|.\|_2}(n) \quad \underbrace{\leq O(n^{3/2-\varepsilon})}_{\text{Zahl '19}}$

Erdős unit distance problem

Question (Erdős, 1946)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^d, \|.\|_2)$?

Let $U_{\|.\|_2}(n)$ denote the answer.

In 2D: $\underbrace{\Omega\left(n^{1+\frac{1}{\log\log n}}\right) \leq}_{\text{Erdős '46}} \quad U_{\|.\|_2}(n) \quad \underbrace{\leq O(n^{4/3})}_{\text{Spencer, Szemerédi, Trotter '84}}$

In 3D: $\underbrace{n^{4/3+o(1)} \leq}_{\text{Erdős '60}} \quad U_{\|.\|_2}(n) \quad \underbrace{\leq O(n^{3/2-\varepsilon})}_{\text{Zahl '19}}$

Erdős unit distance problem

Question (Erdős, 1946)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^d, \|.\|_2)$*?*

Let $U_{\|.\|_2}(n)$ denote the answer.

In 2D: $\underbrace{\Omega\left(n^{1+\frac{1}{\log\log n}}\right) \leq}_{\text{Erdős '46}} \quad U_{||.||_2}(n) \quad \underbrace{\leq O(n^{4/3})}_{\text{Spencer, Szemerédi, Trotter '84}}$

In 3D: $\underbrace{n^{4/3+o(1)} \leq}_{\text{Erdős '60}} \quad U_{||.||_2}(n) \quad \underbrace{\leq O(n^{3/2-\varepsilon})}_{\text{Zahl '19}}$

In 4 and more D: $U_{||.||_2}(n) = \Theta(n^2)$

Erdős unit distance problem in general normed spaces

Let $\|.\|$ be an arbitrary $\mathbb{R}^d$-norm and $U_{\|.\|}(n)$ denote the answer to the following

Question (Erdős, Ulam 1980)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^d, \|.\|)$*?*

Erdős unit distance problem in general normed spaces

Let $\|.\|$ be an arbitrary $\mathbb{R}^d$-norm and $U_{\|.\|}(n)$ denote the answer to the following

Question (Erdős, Ulam 1980)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^d, \|.\|)$*?*

- Folklore: for any norm $U_{\|.\|}(n) \geq (\frac{1}{2} - o(1))n \log_2 n$.

Erdős unit distance problem in general normed spaces

Let $\|.\|$ be an arbitrary $\mathbb{R}^d$-norm and $U_{\|.\|}(n)$ denote the answer to the following

Question (Erdős, Ulam 1980)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^d, \|.\|)$*?*

- Folklore: for any norm $U_{\|.\|}(n) \geq (\frac{1}{2} - o(1))n \log_2 n$.
- **Brass** 1996: Is there an $\mathbb{R}^2$-norm for which $U_{\|.\|}(n) = \Theta(n \log n)$?

Erdős unit distance problem in general normed spaces

Let $\|.\|$ be an arbitrary $\mathbb{R}^d$-norm and $U_{\|.\|}(n)$ denote the answer to the following

Question (Erdős, Ulam 1980)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^d, \|.\|)$*?*

- Folklore: for any norm $U_{\|.\|}(n) \geq (\frac{1}{2} - o(1))n \log_2 n$.
- **Brass** 1996: Is there an $\mathbb{R}^2$-norm for which $U_{\|.\|}(n) = \Theta(n \log n)$?
- **Matoušek** 2011: For "most" $\mathbb{R}^2$-norms $U_{\|.\|}(n) \leq O(n \log n \log \log n)$.

Erdős unit distance problem in general normed spaces

Let $\|.\|$ be an arbitrary $\mathbb{R}^d$-norm and $U_{\|.\|}(n)$ denote the answer to the following

Question (Erdős, Ulam 1980)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^d, \|.\|)$?

- Folklore: for any norm $U_{\|.\|}(n) \geq (\frac{1}{2} - o(1))n \log_2 n$.
- **Brass** 1996: Is there an $\mathbb{R}^2$-norm for which $U_{\|.\|}(n) = \Theta(n \log n)$?
- **Matoušek** 2011: For "most" $\mathbb{R}^2$-norms $U_{\|.\|}(n) \leq O(n \log n \log \log n)$.
- **Brass-Moser-Pach** 2006: For $d \geq 3$ show that $\forall$ $\mathbb{R}^d$-norms $U_{\|.\|}(n) \gg n \log n$

Erdős unit distance problem in general normed spaces

Let $\|.\|$ be an arbitrary $\mathbb{R}^d$-norm and $U_{\|.\|}(n)$ denote the answer to the following

Question (Erdős, Ulam 1980)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^d, \|.\|)$?

- Folklore: for any norm $U_{\|.\|}(n) \geq (\frac{1}{2} - o(1))n \log_2 n$.
- **Brass** 1996: Is there an $\mathbb{R}^2$-norm for which $U_{\|.\|}(n) = \Theta(n \log n)$?
- **Matoušek** 2011: For "most" $\mathbb{R}^2$-norms $U_{\|.\|}(n) \leq O(n \log n \log \log n)$.
- **Brass-Moser-Pach** 2006: For $d \geq 3$ show that $\forall$ $\mathbb{R}^d$-norms $U_{\|.\|}(n) \gg n \log n$

  For $d \geq 4$ is there an $\mathbb{R}^d$-norm s.t. $U_{\|.\|}(n) = o(n^2)$?

Erdős unit distance problem in general normed spaces

Let $\|.\|$ be an arbitrary $\mathbb{R}^d$-norm and $U_{\|.\|}(n)$ denote the answer to the following

Question (Erdős, Ulam 1980)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^d, \|.\|)$?

- Folklore: for any norm $U_{\|.\|}(n) \geq (\frac{1}{2} - o(1))n \log_2 n$.
- **Brass** 1996: Is there an $\mathbb{R}^2$-norm for which $U_{\|.\|}(n) = \Theta(n \log n)$?
- **Matoušek** 2011: For "most" $\mathbb{R}^2$-norms $U_{\|.\|}(n) \leq O(n \log n \log \log n)$.
- **Brass-Moser-Pach** 2006: For $d \geq 3$ show that $\forall$ $\mathbb{R}^d$-norms $U_{\|.\|}(n) \gg n \log n$

  For $d \geq 4$ is there an $\mathbb{R}^d$-norm s.t. $U_{\|.\|}(n) = o(n^2)$?

Theorem (Alon, B., Sauermann, 2023+)

For any $d \geq 2$ for "most" $\mathbb{R}^d$-norms

$$U_{\|.\|}(n) \leq \frac{d}{2} \cdot n \log_2 n.$$

Erdős unit distance problem in general normed spaces

Let $\|.\|$ be an arbitrary $\mathbb{R}^d$-norm and $U_{\|.\|}(n)$ denote the answer to the following

Question (Erdős, Ulam 1980)

What is the maximum number of unit distances defined by n points in $(\mathbb{R}^d, \|.\|)$*?*

- Folklore: for any norm $U_{\|.\|}(n) \geq (\frac{1}{2} - o(1))n \log_2 n$.
- **Brass** 1996: Is there an $\mathbb{R}^2$-norm for which $U_{\|.\|}(n) = \Theta(n \log n)$?
- **Matoušek** 2011: For "most" $\mathbb{R}^2$-norms $U_{\|.\|}(n) \leq O(n \log n \log \log n)$.
- **Brass-Moser-Pach** 2006: For $d \geq 3$ show that $\forall$ $\mathbb{R}^d$-norms $U_{\|.\|}(n) \gg n \log n$

  For $d \geq 4$ is there an $\mathbb{R}^d$-norm s.t. $U_{\|.\|}(n) = o(n^2)$?

Theorem (Alon, B., Sauermann, 2023+)

For any $d \geq 2$ for "most" $\mathbb{R}^d$-norms

$$\frac{d-1-o(1)}{2} \cdot n \log_2 n \leq U_{\|.\|}(n) \leq \frac{d}{2} \cdot n \log_2 n.$$

Rainbow Turán problem for all cycles

Question (Keevash, Mubayi, Sudakov, Verstraëte, 2006)

What is the minimum number of edges in an n-vertex graph which guarantees a rainbow cycle in any proper edge colouring?

Rainbow Turán problem for all cycles

Question (Keevash, Mubayi, Sudakov, Verstraëte, 2006)

What is the minimum number of edges in an n-vertex graph which guarantees a rainbow cycle in any proper edge colouring?

- We need at least $\frac{1}{2}n \cdot \log_2 n$

Rainbow Turán problem for all cycles

Question (Keevash, Mubayi, Sudakov, Verstraëte, 2006)

What is the minimum number of edges in an n-vertex graph which guarantees a rainbow cycle in any proper edge colouring?

- We need at least $\frac{1}{2} n \cdot \log_2 n$
- Das, Lee, Sudakov: $O(n \cdot e^{\sqrt{\log n}})$ suffice

Rainbow Turán problem for all cycles

Question (Keevash, Mubayi, Sudakov, Verstraëte, 2006)

What is the minimum number of edges in an n-vertex graph which guarantees a rainbow cycle in any proper edge colouring?

- We need at least $\frac{1}{2}n \cdot \log_2 n$
- Das, Lee, Sudakov: $O(n \cdot e^{\sqrt{\log n}})$ suffice
- Janzer: $O(n \cdot \log^4 n)$ suffice

Rainbow Turán problem for all cycles

Question (Keevash, Mubayi, Sudakov, Verstraëte, 2006)

What is the minimum number of edges in an n-vertex graph which guarantees a rainbow cycle in any proper edge colouring?

- We need at least $\frac{1}{2}n \cdot \log_2 n$
- Das, Lee, Sudakov: $O(n \cdot e^{\sqrt{\log n}})$ suffice
- Janzer: $O(n \cdot \log^4 n)$ suffice
- Tomon: $O(n \cdot \log^{2+o(1)} n)$ suffice

Rainbow Turán problem for all cycles

Question (Keevash, Mubayi, Sudakov, Verstraëte, 2006)

What is the minimum number of edges in an n-vertex graph which guarantees a rainbow cycle in any proper edge colouring?

- We need at least $\frac{1}{2}n \cdot \log_2 n$
- Das, Lee, Sudakov: $O(n \cdot e^{\sqrt{\log n}})$ suffice
- Janzer: $O(n \cdot \log^4 n)$ suffice
- Tomon: $O(n \cdot \log^{2+o(1)} n)$ suffice
- Janzer, Sudakov; Kim, Lee, Liu, Tran: $O(n \cdot \log^2 n)$ suffice

Rainbow Turán problem for all cycles

Question (Keevash, Mubayi, Sudakov, Verstraëte, 2006)

What is the minimum number of edges in an n-vertex graph which guarantees a rainbow cycle in any proper edge colouring?

- We need at least $\frac{1}{2}n \cdot \log_2 n$
- Das, Lee, Sudakov: $O(n \cdot e^{\sqrt{\log n}})$ suffice
- Janzer: $O(n \cdot \log^4 n)$ suffice
- Tomon: $O(n \cdot \log^{2+o(1)} n)$ suffice
- Janzer, Sudakov; Kim, Lee, Liu, Tran: $O(n \cdot \log^2 n)$ suffice

Theorem (Alon, B., Sauermann, Zakharov, Zamir)

Any properly coloured n-vertex graph with $O(n \log n \log \log n)$ edges contains a rainbow cycle.

Dissociated sets and additive dimension

Definition

Let $(G, +)$ be a group. A subset $S \subseteq G$ is said to be _dissociated_ *if*

$$\sum_{g \in S} \varepsilon_g g = 0$$

for $\varepsilon_g \in \{0, 1, -1\}$ implies $\varepsilon_g = 0$ for all $g \in S$.

Dissociated sets and additive dimension

Definition

Let $(G, +)$ be a group. A subset $S \subseteq G$ is said to be dissociated if

$$\sum_{g \in S} \varepsilon_g g = 0$$

for $\varepsilon_g \in \{0, 1, -1\}$ implies $\varepsilon_g = 0$ for all $g \in S$.

- Very useful in Harmonic analysis and additive number theory

Dissociated sets and additive dimension

Definition

Let $(G,+)$ be a group. A subset $S \subseteq G$ is said to be dissociated if

$$\sum_{g \in S} \varepsilon_g g = 0$$

for $\varepsilon_g \in \{0,1,-1\}$ implies $\varepsilon_g = 0$ for all $g \in S$.

- Very useful in Harmonic analysis and additive number theory
- Dissociated sets play the same role independent sets play in vector spaces

Dissociated sets and additive dimension

Definition

Let $(G, +)$ be a group. A subset $S \subseteq G$ is said to be dissociated if

$$\sum_{g \in S} \varepsilon_g g = 0$$

for $\varepsilon_g \in \{0, 1, -1\}$ implies $\varepsilon_g = 0$ for all $g \in S$.

- Very useful in Harmonic analysis and additive number theory
- Dissociated sets play the same role independent sets play in vector spaces
  - Maximal dissociated sets are spanning

Dissociated sets and additive dimension

Definition

Let $(G,+)$ be a group. A subset $S \subseteq G$ is said to be dissociated if

$$\sum_{g \in S} \varepsilon_g g = 0$$

for $\varepsilon_g \in \{0,1,-1\}$ implies $\varepsilon_g = 0$ for all $g \in S$.

- Very useful in Harmonic analysis and additive number theory
- Dissociated sets play the same role independent sets play in vector spaces
  - Maximal dissociated sets are spanning
  - Maximal dissociated sets have similar sizes

Dissociated sets and additive dimension

Definition

Let $(G,+)$ be a group. A subset $S \subseteq G$ is said to be dissociated if

$$\sum_{g \in S} \varepsilon_g g = 0$$

for $\varepsilon_g \in \{0,1,-1\}$ implies $\varepsilon_g = 0$ for all $g \in S$.

- Very useful in Harmonic analysis and additive number theory
- Dissociated sets play the same role independent sets play in vector spaces
  - Maximal dissociated sets are spanning
  - Maximal dissociated sets have similar sizes

Definition

Let $(G,+)$ be a group. The additive dimension $\dim A$ *of a subset $A \subseteq G$ is the maximum size of a dissociated subset of A.*

A relation between sumset size and additive dimension

Definition

Let $(G, +)$ be a group. The *additive dimension* $\dim A$ *of a subset* $A \subseteq G$ *is the maximum size of a dissociated subset of* A*.*

A relation between sumset size and additive dimension

Definition

Let $(G,+)$ be a group. The additive dimension $\dim A$ *of a subset* $A \subseteq G$ *is the maximum size of a dissociated subset of* A.

Theorem (Sanders, Skhredov, 2007)

Let $(G,+)$ be an Abelian group and $A \subseteq G$ such that $|A+A| \leq K|A|$ then

$$\dim A \leq O(K \log |A|).$$

A relation between sumset size and additive dimension

Definition

Let $(G,+)$ be a group. The additive dimension $\dim A$ *of a subset $A \subseteq G$ is the maximum size of a dissociated subset of A.*

Theorem (Sanders, Skhredov, 2007)

Let $(G,+)$ be an Abelian group and $A \subseteq G$ such that $|A+A| \leq K|A|$ then

$$\dim A \leq O(K \log |A|).$$

Theorem (Alon, B., Sauermann, Zakharov, Zamir)

Let $(G,\cdot)$ be any group and $A \subseteq G$ such that $|A \cdot A| \leq K|A|$ then

$$\dim A \leq O(K \log |A| \log \log |A|).$$


thank you
danke
謝謝
ngiyabonga
teşekkür ederim
gracias
merci
obrigado
dziękuję
bedankt
спасибо
рахмат
Баярлалаа
faafetai lava
vinaka
mersi
barka
welalin
tack
dank je
misaotra
matondo
paldies
grazzi
mahalo
tapadh leat
хвала
asante
manana
obrigada
tenki
murakoze
mochchakkeram
djiere dieuf
tau
mamnun
дякую
go raibh maith agat
chokrane
trugarez
dakujem
takk
arigatô
shukriya
mercé
мерси
dhanyavadagalu
diolch
grazie
tanemirt
rahmet
taiku
sulpáy
xièxie
ευχαριστώ
감사합니다
ありがとう
terima kasih
kop khun krap
sukriya
sagolun
najis tuke
kam sah hamnida
rahmat
didi madloba
mèsi
dèkuji
sobodi
enkosi
bayarlalaa
gràcie
hvala
maururu
köszönöm
dankie
kiitos
nanni
nandri
dhanyavad
спасибі
blagodaram
kia ora
spas
akun
dankon
aciu
chnorakaloutioun
gratias ago
gràcies
তোমাকে ধন্যবাদ
תודה רבה