Robust sublinear expanders

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Expander graphs

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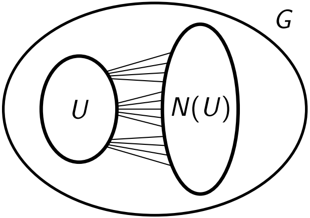
Definition (Expansion)

An n-vertex graph G is a λ-expander if for all $U \subseteq V(G)$ s.t. $|U| \leq \frac{n}{2\lambda}$ we have:

$$|N(U)| > \lambda |U|.$$

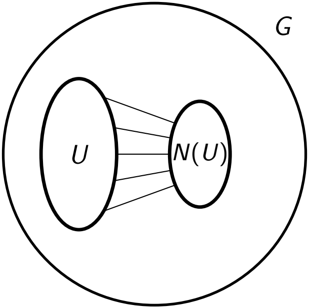


Sublinear expansion

Definition (Sublinear expansion)

An n-vertex graph G is a sublinear expander if $\forall U \subseteq V(G)$ *s.t.* $|U| \leq \frac{n}{2}$ *we have:*

$$|N(U)| > \frac{1}{\log^2 n} \cdot |U|$$



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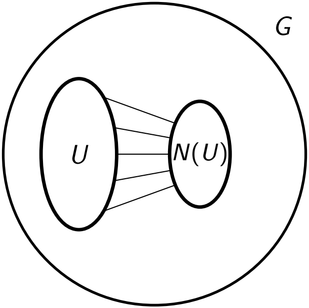
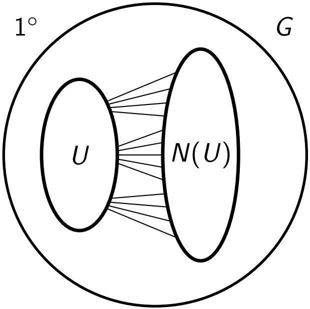


- Komlós and Szemerédi: Can find a sublinear expander of almost the same average degree in any graph.

Definition (Robust sublinear expansion)

An n-vertex graph G is a robust sublinear expander if $\forall U \subseteq V(G): |U| \leq \frac{n}{2}$

$$1^\circ: \quad |N(U)| > \log^2 n \cdot |U| \quad \textit{or}$$



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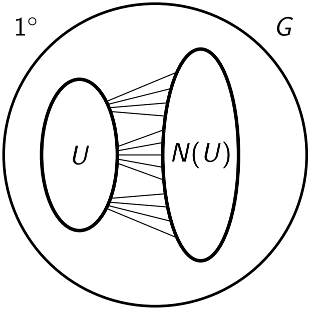
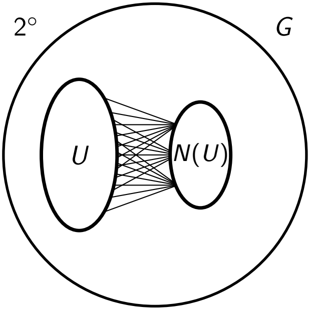
$$1^\circ: \quad |N(U)| > \log^2 n \cdot |U| \quad \textit{or}$$

$$2^\circ: \quad N(U) \textit{ has } > \frac{|U|}{\log^2 n} \textit{ vtcs with } \geq \log^4 n \textit{ neighbours in } U.$$





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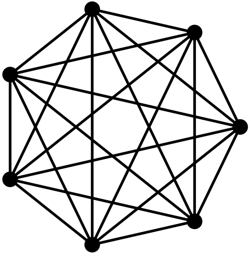

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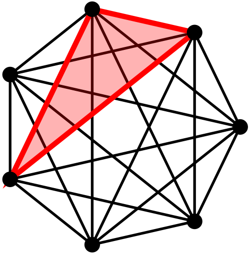

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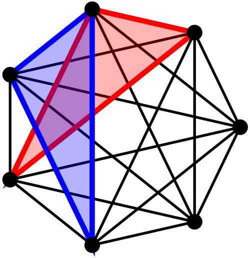

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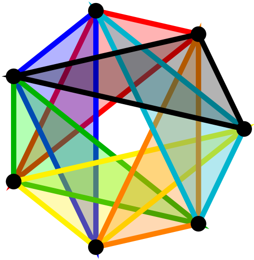

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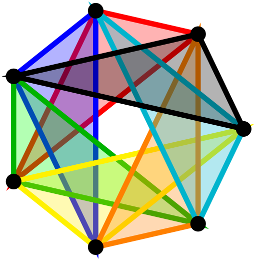

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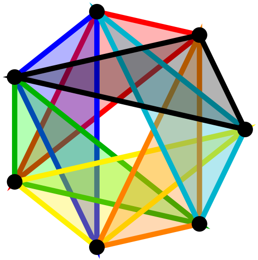

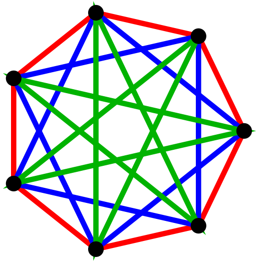

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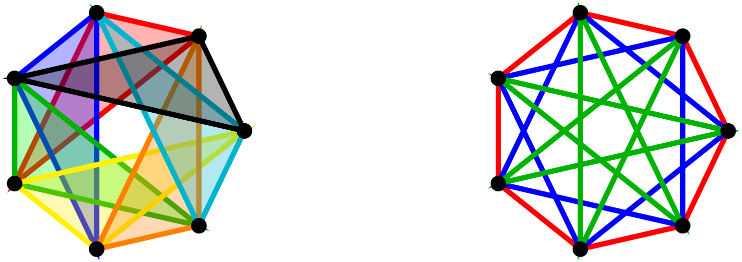

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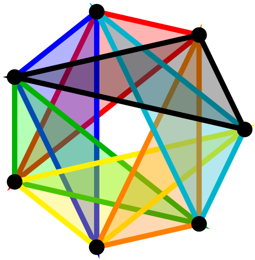

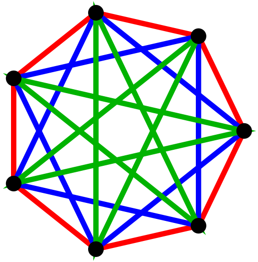

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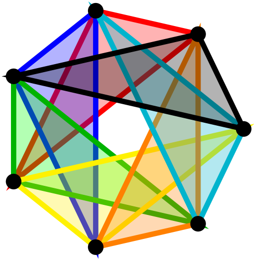

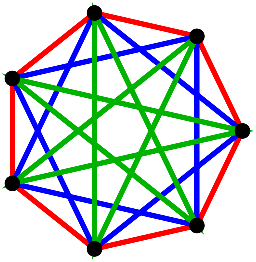

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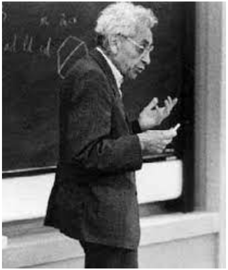
- Can we decompose any graph into cycles? No if $\exists$ an odd degree vertex.
- Veblen 1912: Any graph with all degrees even decomposes into cycles.

Erdős-Gallai Conjecture

Conjecture (Erdős-Gallai 1960s)

Every n-vertex graph can be decomposed into $O(n)$ cycles and edges.



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Erdős-Gallai Conjecture: related work

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- Lovász 1968: True for paths in place of cycles

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Theorem (B., Montgomery 2022+)

Any n-vertex graph can be decomposed into $O(n \log^\star n)$ cycles and edges.

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In 4 and more D: $$U_{||.||_2}(n) = \Theta(n^2)$$

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Let $\|.\|$ be an arbitrary $\mathbb{R}^d$-norm and $U_{\|.\|}(n)$ denote the answer to the following

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Theorem (Alon, B., Sauermann, 2023+)

For any $d \geq 2$ *for "most"* $\mathbb{R}^d$*-norms*

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Rainbow Turán problem for all cycles

Question (Keevash, Mubayi, Sudakov, Verstraëte, 2006)

What is the minimum number of edges in an n-vertex graph which guarantees a rainbow cycle in any proper edge colouring?

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Theorem (Alon, B., Sauermann, Zakharov, Zamir)

Any properly coloured n-vertex graph with $O(n \log n \log\log n)$ edges contains a rainbow cycle.

Dissociated sets and additive dimension

Definition

Let $(G, +)$ be a group. A subset $S \subseteq G$ is said to be <u>*dissociated*</u> *if*

$$\sum_{g \in S} \varepsilon_g g = 0$$

for $\varepsilon_g \in \{0, 1, -1\}$ implies $\varepsilon_g = 0$ for all $g \in S$.

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Theorem (Alon, B., Sauermann, Zakharov, Zamir)

Let $(G, \cdot)$ *be any group and* $A \subseteq G$ *such that* $|A \cdot A| \leq K|A|$ *then*

$$\dim A \leq O(K \log |A| \log \log |A|).$$


thank you
gracias
danke
merci
obrigado
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