Towards the Erdős-Gallai Cycle Decomposition Conjecture

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joint work with Richard Montgomery

Graph decomposition problems

- General graph decomposition question:

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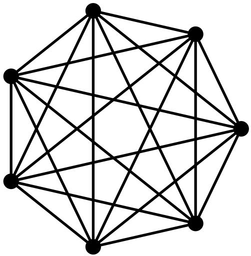

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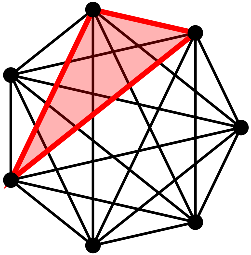

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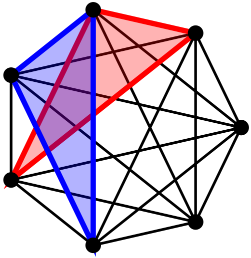

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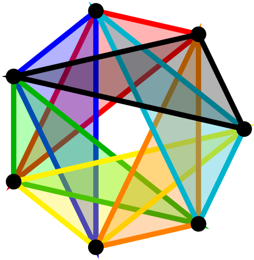

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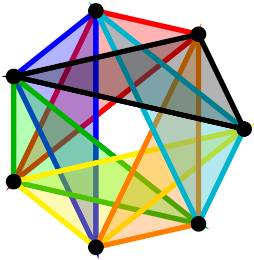

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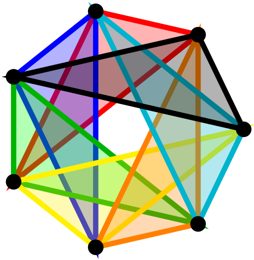

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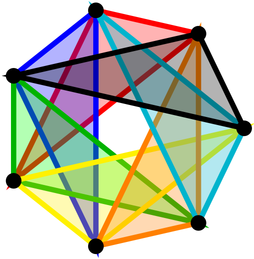

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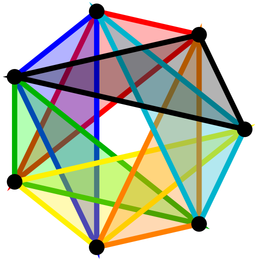

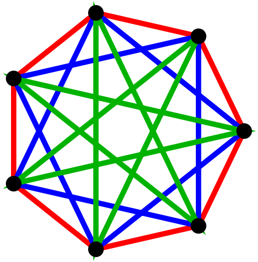

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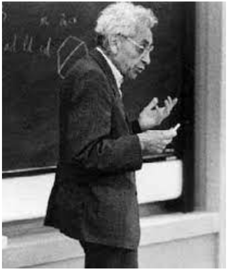
Conjecture (Erdős-Gallai 1960s)

Every n-vertex graph can be decomposed into $O(n)$ cycles and edges.



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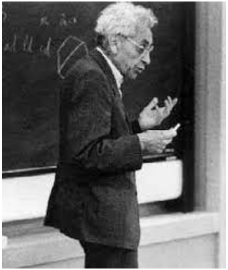

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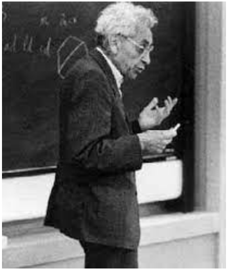

Conjecture (Hajós 1960s)

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- Pyber 1985: Precise solution for the covering version.

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Theorem (B., Montgomery 2022+)

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Path decompositions with well-spread endvertices

Theorem (Lovász, 1968)

Every n-vertex graph can be decomposed into at most $\frac{n}{2}$ paths and cycles.

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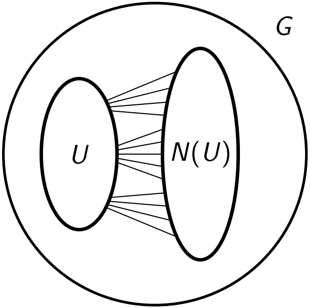
- n odd degree vertices $\implies$ it is a path decomposition and
- each original vertex is an endpoint of exactly one path.
- Removing v gives the desired decomposition.

Robust sublinear expansion

Definition (Expansion)

An n-vertex graph G is an expander if for all $U \subseteq V(G)$ s.t. $|U| \leq \frac{n}{2F}$ we have:

$$|N(U)| > F|U|.$$



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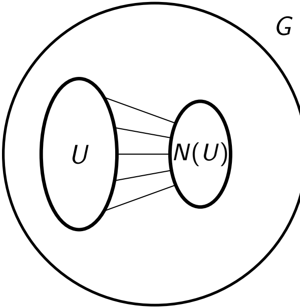

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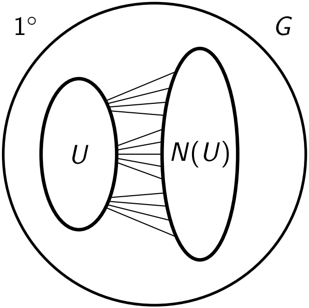

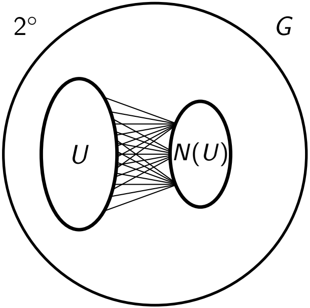

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    - such that $|H_1| + \ldots + |H_t| \leq 2n$

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    - Can be used to "parallellize" some of our arguments.

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  - Existence of an expanding "skeleton".

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- We prove a number of properties of this weaker notion of expander graphs:
  - An (almost) partitioning of an arbitrary graph into expanders lemma.
  - An edge-subsampling lemma.
  - A very strong connectivity property even under vertex subsampling
  - Existence of an expanding "skeleton".

Proof sketch.

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Any n-vertex graph can be decomposed into $6n$ cycles and $O(n \log^C n)$ edges.

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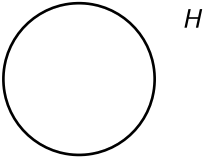

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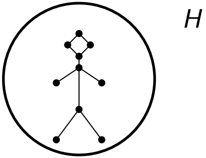

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Theorem (B., Montgomery 2022+)

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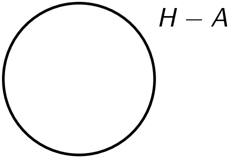

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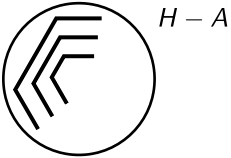

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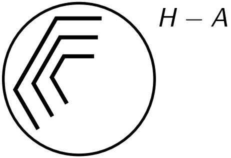

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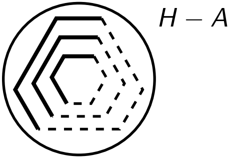

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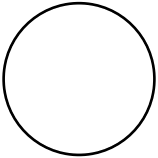

- Split A into expanding skeletons A_1, A_2, A_3

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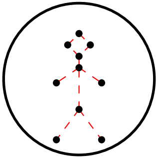

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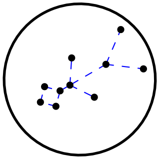

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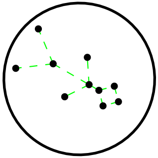

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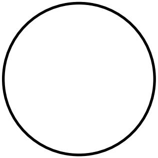

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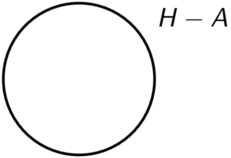

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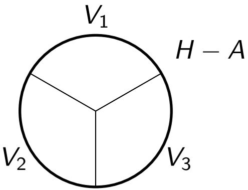

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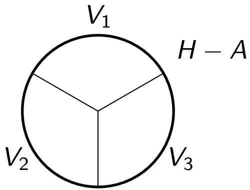

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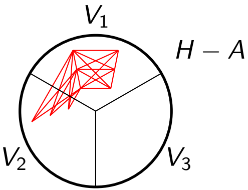

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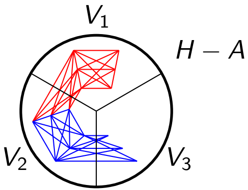

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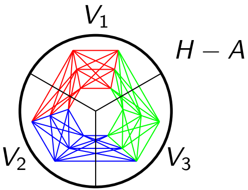

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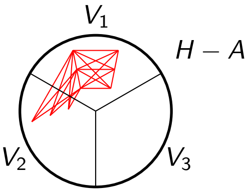

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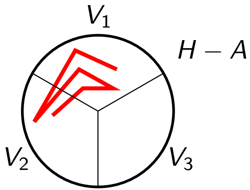

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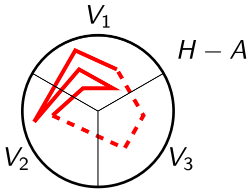

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Iteration.

Theorem (B., Montgomery 2022+)

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Iteration.

Theorem (B., Montgomery 2022+)

Any n-vertex graph can be decomposed into $6n$ cycles and $O(n \log^C n)$ edges.

- Step 1: Remove the $6n$ cycles to get a leftover graph with $O(n \log^C n)$ edges

Iteration.

Theorem (B., Montgomery 2022+)

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Theorem (B., Montgomery 2022+)

Any n-vertex graph can be decomposed into $O(n)$ cycles and $O(n \log^C \log n)$ edgs

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For any constant k any n-vertex graph can be decomposed into $O(kn)$ cycles and $O(n \underbrace{\log \log \ldots \log}_{k} n)$ edges.

Concluding remarks

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Any n-vertex graph can be decomposed into $O(n \log^\star n)$ cycles and edges.

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Any Eulerian n-vertex graph can be decomposed into $O(n\log^\star n)$ cycles.

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Conjecture (Erdős-Gallai, 1960s)

Every n-vertex graph can be decomposed into $O(n)$ cycles and edges.

рахмат
danke
謝謝
ngiyabonga
spas
teşekkür ederim
Баярлалаа
mersi
kia ora
barka
welalin
tack
спасибо
faafetai lava
vinaka
спасибі
blagodaram
dank je
misaotra
matondo
paldies
grazzi
mahalo
kiitos
dankie
thank you
gracias
tapadh leat
хвала
asante
manana
obrigada
tenki
bedankt
nanni
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dhanyavad
enkosi
bayarlalaa
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তোমাকে ধন্যবাদ
감사합니다
xièxie
ευχαριστώ
merci
תודה רבה