Towards the Erdős-Gallai Cycle Decomposition Conjecture

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Institute for Advanced Study and Princeton University

joint work with Richard Montgomery

Graph decomposition problems

- General graph decomposition question:

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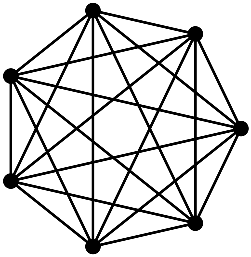

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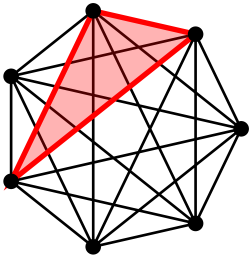

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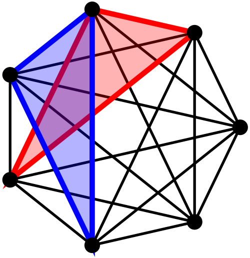

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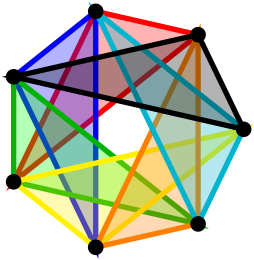

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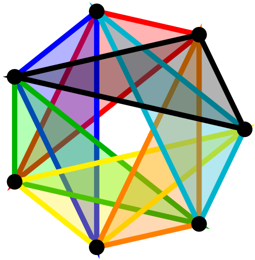

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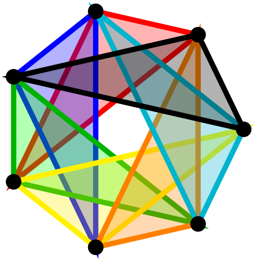

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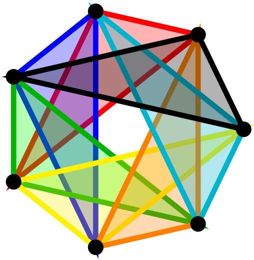

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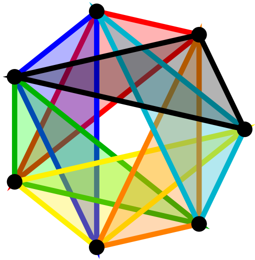

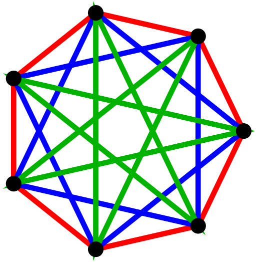

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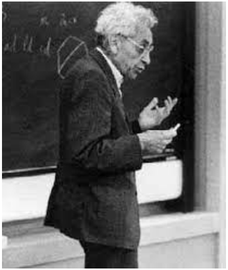
Every n-vertex graph can be decomposed into $O(n)$ cycles and edges.



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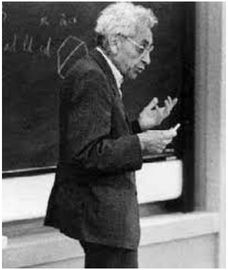

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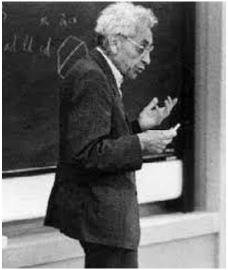

Conjecture (Hajós 1960s)

Every Eulerian n-vertex graph can be decomposed into at most $\lfloor n/2 \rfloor$ cycles.

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Any n-vertex graph can be decomposed into $O(n \log^\star n)$ cycles and edges.

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Definition (Expansion)

An n-vertex graph G is an expander if for all $U \subseteq V(G)$ we have:

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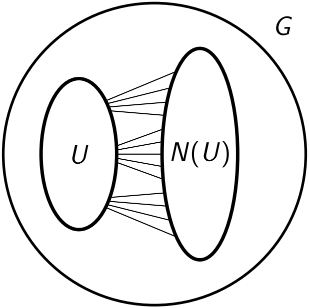

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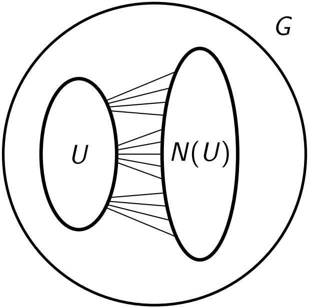

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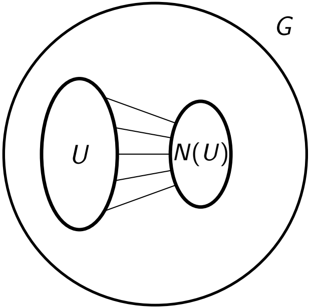

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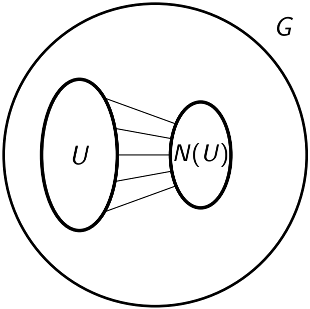

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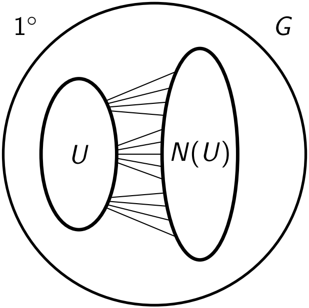

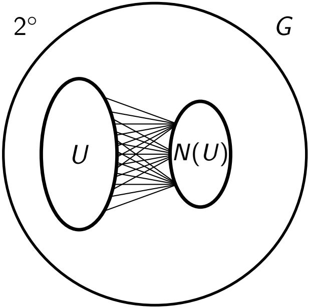

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  - A vertex-subsampling lemma.
  - Existence of an expanding "skeleton".

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Theorem (Lovász, 1968)

Every n-vertex graph can be decomposed into at most $\frac{n}{2}$ paths and cycles.

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- Apply Lovász' Theorem.

Proof sketch.

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Any n-vertex graph can be decomposed into $O(n)$ cycles and $O(n \log^C n)$ edges.

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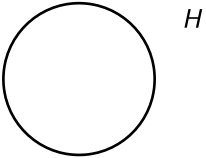

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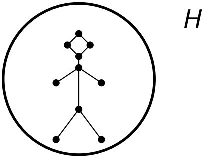

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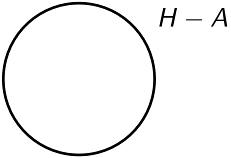

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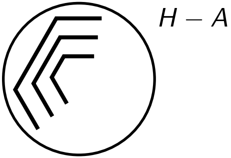

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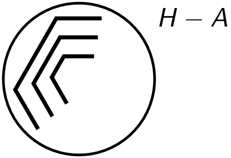

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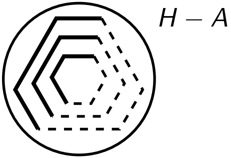

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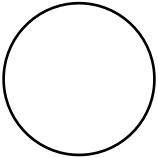

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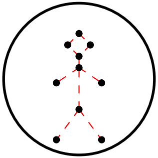

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Any n-vertex graph can be decomposed into $O(n)$ cycles and $O(n \log^C n)$ edges.

Proof:

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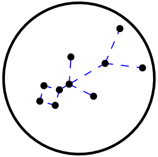

- Split A into expanding skeletons A_1, A_2, A_3

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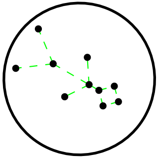

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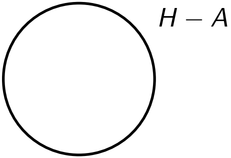

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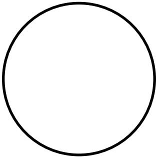

$H - A$

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Proof sketch.

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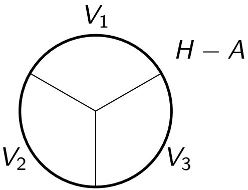

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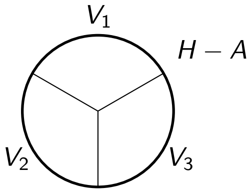

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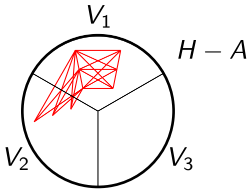

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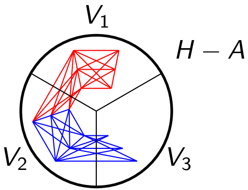

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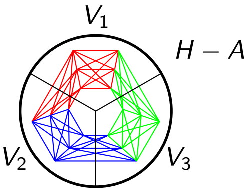

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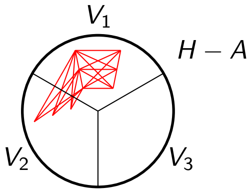

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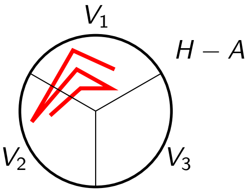

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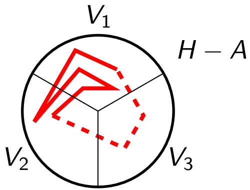

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Concluding remarks

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Theorem (B., Montgomery 2022+)

For any constant k any n-vertex graph can be decomposed into $O(n)$ cycles and $O(n\underbrace{\log\log\ldots\log}_{k} n)$ edges.

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Any n-vertex graph can be decomposed into $O(n\log^{\star} n)$ cycles and edges.

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Any Eulerian n-vertex graph can be decomposed into $O(n \log^\star n)$ cycles.

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Conjecture (Erdős-Gallai, 1960s)

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рахмат
danke
謝謝
ngiyabonga
Баярлалаа
spas
teşekkür ederim
mersi
kia ora
barka
welalin
tack
спасибо
faafetai lava
vinaka
dank je
misaotra
matondo
paldies
grazzi
mahalo
tapadh leat
kiitos
dankie
спасибі
blagodaram
thank you
gracias
хвала
nanni
nandri
dhanyavad
mauruuru
köszönöm
aciū
asante
manana
obrigada
tenki
bedankt
bayarlalaa
gràcie
hvala
akun dankon
djiere dieuf
tau
mochchakkeram
murakoze
enkosi
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gratias ago
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go raibh maith agat
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ありがとう
tanemirt
grazie
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takk
dakujem
trugarez
didi madloba
najis tuke
terima kasih
rahmet
diolch
dhanyavadagalu
shukriya
mercê
мерси
kam sah hamnida
rahmat
তোমাকে ধন্যবাদ
감사합니다
xièxie
ευχαριστώ
merci
תודה רבה