Math 205-The Fargues-Fontaine Curve

September 28, 2018

Several years ago, Fargues and Fontaine discovered a remarkable geometric object $X$, which plays an essential role in $p$-adic analytic geometry. This object, now known the Fargues-Fontaine curve, has close connections with local class field theory, $p$-adic Hodge theory, the theory of perfectoid spaces, and the local Langlands program. The goal of this course is to define the Fargues-Fontaine curve $X$, to establish its basic properties, and to understand the classification of vector bundles on $X$. If time permits, we may delve deeper into other aspects of the subject (depending on the interests of the audience).

Instructor: Jacob Lurie (lurie@math.harvard.edu). Office hours by appointment.

Time/Place: MWF 9:00-9:50, APM 5402.

Course Website: http://www.math.harvard.edu/~lurie/205.html

Likely Topics:
- Perfectoid fields
- Tilting
- Period rings
- The "fundamental exact sequence" of $p$-adic Hodge theory
- The Fargues-Fontaine curve and its properties
- Classification of vector bundles on the Fargues-Fontaine curve

Possible further topics:
- $p$-divisible groups and vector bundles on the Fargues-Fontaine curve
- $p$-adic Hodge theory
- The Colmez-Fontaine theorem
- The twistor space analogy
- Local class field theory via the Fargues-Fontaine curve
- Banach-Colmez spaces