Math 155, Problem Set 7 (due October 31)

October 22, 2011

(1) Let S be a set of size $2m$. Show that the partially ordered set $P(S) = \{T : T \subseteq S\}$ has a unique antichain of size $\binom{2m}{m}$.

(2) Let A be a partially ordered set, and let m and n be integers. Show that if A has more that mn elements, then either A has a chain of size $m+1$ or an antichain of size $n+1$.

(3) Let A be a finite partially ordered set, and let $a, b \in A$ be elements. Show that the following conditions are equivalent:

(i) We have $a \leq b$ in A.

(ii) For every linear ordering $\leq'$ on A which refines $\leq$, we have $a \leq' b$.