# Math 155, Problem Set 5 (due October 17)

October 10, 2011

(1) Up to rotational symmetry, how many ways are there to color the faces of a regular dodecahedron using two colors?

(2) Let $S$ be the species of derangements (so that, for every finite set $I$, $S[I]$ is the set of permutations of $I$ without fixed points). Find a formula for the cycle index series $Z_S(s_1, s_2, \ldots)$.

Let $G$ be a finite group. Recall that the Burnside ring $\mathrm{Burn}[G]$ is generated by symbols $[X]$, where $X$ is a finite $G$-set, modulo the following relations:

- If $X$ and $Y$ are isomorphic $G$-sets, then $[X] = [Y]$.
- For every pair of finite $G$-sets $X$ and $Y$, $[X \amalg Y] = [X] + [Y]$.

(3) Let $\{H_i\}_{1 \leq i \leq m}$ be a collection of representatives for the conjugacy classes of subgroups of $G$ (so that every subgroup $H \subseteq G$ is conjugate to $H_i$ for some unique $i$). Show that, as an abelian group, $\mathrm{Burn}[G]$ is freely generated by the elements $[H_i \backslash G]$. That is, show that every element of $\mathrm{Burn}[G]$ can be written uniquely as a sum

$$\sum_{1 \leq i \leq m} c_i [H_i \backslash G],$$

for some integers $c_i$.