Syllabus

MATH 83N: Proofs and Modern Mathematics

Fall 2019

Official Course Description:
How do mathematicians think? Why are the mathematical facts learned in school true? In this course students will explore higher-level mathematical thinking and will gain familiarity with a crucial aspect of mathematics: achieving certainty via mathematical proofs, a creative activity of figuring out what should be true and why. This course is ideal for students who would like to learn about the reasoning underlying mathematical results, but at a pace and level of abstraction not as intense as Math 61CM/DM, as a consequence benefiting from additional opportunity to explore the reasoning. Familiarity with one-variable calculus is strongly recommended at least at the AB level of AP Calculus since a significant part of the seminar develops develops some of the main results in that material systematically from a small list of axioms. We also address linear algebra from the viewpoint of a mathematician, illuminating notions such as fields and abstract vector spaces.

Time: Monday, Wednesday 1:30pm – 2:50pm

Location: Gilbert Biological Sciences Building, Room 117

Instructor: Lisa Sauermann
Email: `lsauerma@stanford.edu`

Course Assistant: Adva Wolf
Email: `advaw@stanford.edu`

Office hours:
Lisa Sauermann: Friday 1:00pm – 3:00pm, Building 380, Room 382-H
Adva Wolf: Tuesday 4:00pm – 6:00pm, Building 380, Room 381-L

Textbooks:
There are two (required) course texts for this class:

- *Linear Algebra Done Right* by Sheldon Axler (3rd edition, ISBN: 0387982582)
- *Understanding Analysis* by Stephen Abbott (2nd edition, ISBN: 1493927116)

As Stanford students, you have *free* online access to these books via Stanford's library website (and you can download the pdf). You can also buy hard copies of the book, but this is *not* required for the class.

We will only cover a small part of each of these books (see the schedule below for more details).

A supplemental book on reading and writing mathematical proofs is the following.

- *How to Read and Do Proofs: An Introduction to Mathematical Thought Processes* by Daniel Solow (6th edition, ISBN: 1118164024)

This book is not required and will not be used it this class, but you may find it helpful to read some sections of this book (in particular the section on quantifiers). A hard-copy of the

book is available through Stanford's library, and the previous edition of the book can also be accessed electronically through Stanford's library website.

Canvas:
Homework and assignments will be posted on canvas (`canvas.stanford.edu`).

Grading policy:
The grades are mostly based on the weekly homeworks and on two writing projects. A smaller part of the grade will be based on the quizzes at the start of every class, and a short presentation. The precise break-down is as follows:

- Homework: 50% (lowest score dropped)
- Linear Algebra Project: 20%
- Real Analysis Project: 20%
- Daily Quizzes: 5%
- Presentation: 5%

Homework:
There will be weekly homework problem set, due in class every Monday. You have to submit the homework on paper and stapled. For most of the homeworks, you are welcome to either hand-write your solutions or to type them up and print them out (there will only be one exceptional week in which you have to type your homework up, in order to practice typing math before you start working on the first writing project). If you hand-write your homework solutions, they must be readable (you won't receive credit for work that the grader can't read).

Your lowest homework score will be dropped to accommodate unforseen circumstances (e.g. illness). Late homework will not be accepted, and no make-up homework will be given.

The homework problems will have a range in difficulty and you should not expect to be able to solve the problems right away. If you are stuck on a problem for a while, you are more than welcome to ask for help in office hours.

You are encouraged to work together with each other on the homework, for example in study groups. I recommend that you first think about each problem yourself for a bit before discussing it with others (I believe that this way you will understand the problem better and learn more from it). In any case, you are required to write down the solution yourself in your own words.

Writing projects:
While the homework will give you lots of practice in writing short mathematical proofs, the writing projects give you the opportunity to write a more difficult proof consisting of several steps (with intermediate lemmas leading up to the proof of the full result).

There will be two writing projects, one on linear algebra and one on real analysis. For each of them, you will write about a given topic that is not covered in class. This means that you will first need to read about the topic, and then write a short expository paper on it (roughly 3 pages long). You will receive detailed instructions on the content and structure of each paper.

You need to type both of these writing projects and submit them printed out on paper (and stapled).

Presentations:
While this course is mostly focusing on writing proofs, it is also an important skill to present proofs at the blackboard. Each student will give a short presentation (around 5 minutes long) of the solution of a homework problem once during the quarter. Each week, some homework problem(s) will be marked to be presented during the Wednesday class in the week the homework is due. You will sign up for the date of your presentation during the second week of class (make sure to keep a copy or take a photo of your solution before handing it in two days before your presentation).

Quizzes:
At the start of each class, there will be a short quiz (at most 5 minutes long) about the material of the previous class. The quiz will ask you to reproduce an important definition or statement from the last class and/or to answer apply such a definition or statement to a basic example. In order to learn mathematics, it is crucial to remember definitions and statements (and to understand what they mean). It is therefore important that you review the material of each class before the start of the next class.

Class structure:
As mentioned above, each class will start with a short quiz on the material of the previous class. On Wednesdays, the quiz will be followed by one or two presentations of homework solutions (also see above). The rest of the class time will a lecture with several short periods of time allotted for students to work on examples and small problems (individually and/or in pairs or small groups).

If you have to miss a class, you are responsible for asking a fellow students for their class notes and any other relevant information given in that class.

Schedule:
A tentative schedule for the class is as follows (this schedule is subject to change):

- Week 1: Sets, functions, and fields (Abbott Chapter 1.1–2, handout).
- Week 2: Vector spaces and subspaces (Axler Chapter 1.B–C).
- Week 3: Span and Linear independence (Axler Chapter 2.A).
- Week 4: Bases and dimension (Axler Chapter 2.B–C).
- Week 5: Linear maps (Axler Chapter 3.A–B).
- Week 6: Matrices (Axler Chapter 3.C).
- Week 7: Real numbers, sequences and limits (Abbott Chapters 1.3–4, 2.2).
- Week 8: Convergence theorems (Abbott Chapter 2.3–5).
- Week 9: Continuous functions (Abbott Chapters 3.2, 4.2–5)
- Week 10: Derivatives and the Mean Value Theorem (Abbott Chapter 5.1–3).

If you have questions about (or are confused by) the material in class or the homework, you are encouraged to come to office hours (see above).

If you have any issues that you would like to discuss in private, please email the instructor to set up a meeting.

Your mental health is very important. There are many different resources available, see `https://mentalhealth.stanford.edu/` for an overview.

Course-related expenses:
All students should retain receipts for books and other course-related expenses, as these may be qualified educational expenses for tax purposes. If you are an undergraduate receiving financial aid, you may be eligible for additional financial aid for required books and course materials if these expenses exceed the aid amount in your award letter. For more information, review your award letter or visit the Student Budget website:
`https://financialaid.stanford.edu/undergrad/budget/index.html`.

Students with Documented Disabilities:
Students who may need an academic accommodation based on the impact of a disability must initiate the request with the Office of Accessible Education (OAE). Professional staff will evaluate the request with required documentation, recommend reasonable accommodations, and prepare an Accommodation Letter for faculty. Unless the student has a temporary disability, Accommodation letters are issued for the entire academic year. Students should contact the OAE as soon as possible since timely notice is needed to coordinate accommodations. The OAE is located at 563 Salvatierra Walk (phone: 723-1066, URL: `https://oae.stanford.edu/`).

Hume Center:
The Hume Center for Writing and Speaking offers free individual appointments and drop-in tutoring. More information can be found on the Hume Center website:
`http://hume.stanford.edu`.

Academic Integrity:
The Honor Code articulates Stanford University's expectations of students and faculty in establishing and maintaining the highest standards in academic work. Examples of conduct that have been regarded as being in violation of the Honor Code (and are most relevant for this course) include copying from another student's work or allowing another student to copy from your own work; plagiarism; revising and resubmitting your work for regrading without the instructor's knowledge and consent; representing as one's own work the work of another. See `http://communitystandards.stanford.edu/` for more information on the Honor Code.

Important Dates:
First Day of Classes ..... September 23
Add/Drop Deadline ..... October 11
Course Withdrawal & Change of Grading Basis Deadlines ..... November 15
Thanksgiving Recess ..... November 25–29
Last Day of Classes, Last Day to Arrange an Incomplete ..... December 6