Syllabus

MATH 233A: Algebraic Methods in Extremal Combinatorics

Fall 2019

Course Description:
MATH233A is a graduate topics course on combinatorics. In Fall 2019, the topic is "Algebraic Methods in Extremal Combinatorics". The class covers different algebraic methods used in extremal combinatorics. Some of the results discussed in the course are a few decades old, while others are as recent as this year.

Time: Monday, Wednesday, Friday 9:30am – 10:20am

Location: Building 200, Room 30 (in the basement)

Instructor: Lisa Sauermann
Email: `lsauerma@stanford.edu`

Office hours:
Lisa Sauermann: Thursday 9:15am – 11:15am, Building 380, Room 382-H

Course website: `http://stanford.edu/~lsauerma/233`

Homework: There will be three problem sets over the course of the quarter.

Please turn page.

Schedule:
A tentative schedule for the class is as follows (this schedule is subject to change):

- Week 1: Linear independence and rank methods (Oddtown theorem, Fisher's inequality, equiangular lines, point-sets with only two distances). [M, Miniatures 3,4,9,15]
- Week 2: more linear algebra methods (counterexamples to Borsuk's conjecture, Bollobás's Theorem). [M, Miniatures 17,18,33]
- Week 3: Polynomial methods relying on the number of zeros of a one-variable polynomial (the Finite Field Kakeya Problem, the Finite Field Nikodym Problem, the Joints Problem). [M, Miniature 25], [G, Chapter 2]
- Week 4: Spectral Methods (the Friendship Theorem, expanders, Huang's proof of the Sensitivity Conjecture). [13, pp. 55–57], [3, Section 9.2], [11]
- Week 5: the Combinatorial Nullstellensatz (and different applications, including the Chevalley-Warning-Theorem, the Erdős-Ginzburg-Ziv theorem and Reiher's proof of Kemnitz' conjecture). [1, 14]
- Week 6: Alon and Dubiner's result proving linearity of the Erdős-Ginzburg-Ziv-constant $\mathfrak{s}(\mathbb{F}_p^d)$ for fixed d. [2]
- Week 7: the Croot-Lev-Pach polyomial method (and different applications, including Ellenberg and Gijswijt's bound for the cap-set problem, the multi-colored sum-free problem, and bounds for sunflower-free sets). [7, 8, 12, 16]
- Week 8: more applications of the Croot-Lev-Pach polynomial method and its consequences (the arithmetic triangle removal lemma, and upper bounds for the Erdős-Ginzburg-Ziv-constant $\mathfrak{s}(\mathbb{F}_p^d)$ for fixed p). [9, 10, 15]
- Week 9: Lower bounds for extremal numbers of graphs: explicit constructions coming from algebraic geometry, and randomized algebraic constructions of Bukh and Conlon. [4, 5, 6]
- Week 10: TBD.

Important Dates:

First Day of Classes	September 23
Add/Drop Deadline	October 11
Course Withdrawal & Change of Grading Basis Deadlines	November 15
Thanksgiving Recess	November 25–29
Last Day of Classes, Last Day to Arrange an Incomplete	December 6

References:
There are no required materials for this course. The course does not follow any particular textbook, but for the first three weeks, we will cover some parts of the following two books:

[G] L. Guth, *Polynomial methods in combinatorics*, University Lecture Series, 64. American Mathematical Society, ISBN: 978-1-4704-2890-7.

[M] J. Matoušek, *Thirty-three Miniatures: Mathematical and Algorithmic Applications of Linear Algebra*, Student Mathematical Library, 53, American Mathematical Society, ISBN: 978-0-8218-4977-4.
A free electronic version is available here:
`http://kam.mff.cuni.cz/~matousek/stml-53-matousek-1.pdf`

Here is the list of references for the remaining seven weeks of the course:

[1] N. Alon, *Combinatorial Nullstellensatz*, Combin. Probab. Comput. **8** (1999), 7–29.

[2] N. Alon and M. Dubiner, *A lattice point problem and additive number theory*, Combinatorica **15** (1995), 301–309.

[3] N. Alon and J. H. Spencer, *The probabilistic method*, Fourth edition, Wiley Series in Discrete Mathematics and Optimization, ISBN: 978-1-119-06195-3.

[4] B. Bukh, *Random algebraic construction of extremal graphs*, Bull. London Math. Soc. **47** (2015), 939–945.

[5] B. Bukh and D. Conlon, *Rational exponents in extremal graph theory*, J. Eur. Math. Soc. **20** (2018), 1747–1757.

[6] D. Conlon, *Graphs with few paths of prescribed length between any two vertices*, preprint, 2014, arXiv:1411.0856.

[7] E. Croot, V. Lev, and P. Pach, *Progression-free sets in $\mathbb{Z}_4^n$ are exponentially small*, Ann. of Math. (2) **185** (2017), 331–337.

[8] J. Ellenberg and D. Gijswijt, *On large subsets of $\mathbb{F}_q^n$ with no three-term arithmetic progression*, Ann. of Math. (2) **185** (2017), 339–343.

[9] J. Fox and L. M. Lovász, *A tight bound for Green's arithmetic triangle removal lemma in vector spaces*, Adv. Math. **321** (2017), 287–297.

[10] J. Fox and L. Sauermann, *Erdős-Ginzburg-Ziv constants by avoiding three-term arithmetic progressions*, Electron. J. Combin. **25** (2018), Paper 2.14, 9 pp.

[11] H. Huang, *Induced subgraphs of hypercubes and a proof of the Sensitivity Conjecture*, to appear in Ann. of Math., preprint, 2019, arXiv:1907.00847.

[12] E. Naslund and W. Sawin, *Upper bounds for sunflower-free sets*, Forum Math. Sigma **5** (2017), e15, 10pp.

[13] B. Nica, *A brief introduction to Spectral Graph Theory*, expanded lecture notes, 2016, arXiv:1609.08072.

[14] C. Reiher, *On Kemnitz' conjecture concerning lattice-points in the plane*, Ramanujan J. **13** (2007), 333–337.

[15] L. Sauermann, *On the size of subsets of $\mathbb{F}_p^n$ without p distinct elements summing to zero*, preprint, 2019, arXiv:1904.09560.

[16] T. Tao, *A symmetric formulation of the Croot–Lev–Pach–Ellenberg–Gijswijt capset bound*, blog post, 2016, `http://terrytao.wordpress.com/2016/05/18/a`.

Your mental health is very important. There are many different resources available, see https://mentalhealth.stanford.edu/ for an overview.

Course-related expenses:
All students should retain receipts for books and other course-related expenses, as these may be qualified educational expenses for tax purposes. If you are an undergraduate receiving financial aid, you may be eligible for additional financial aid for required books and course materials if these expenses exceed the aid amount in your award letter. For more information, review your award letter or visit the Student Budget website:
https://financialaid.stanford.edu/undergrad/budget/index.html.

Students with Documented Disabilities:
Students who may need an academic accommodation based on the impact of a disability must initiate the request with the Office of Accessible Education (OAE). Professional staff will evaluate the request with required documentation, recommend reasonable accommodations, and prepare an Accommodation Letter for faculty. Unless the student has a temporary disability, Accommodation letters are issued for the entire academic year. Students should contact the OAE as soon as possible since timely notice is needed to coordinate accommodations. The OAE is located at 563 Salvatierra Walk (phone: 723-1066, URL: https://oae.stanford.edu/).

Hume Center:
The Hume Center for Writing and Speaking offers free individual appointments and drop-in tutoring. More information can be found on the Hume Center website:
http://hume.stanford.edu.

Academic Integrity:
The Honor Code articulates Stanford University's expectations of students and faculty in establishing and maintaining the highest standards in academic work. Examples of conduct that have been regarded as being in violation of the Honor Code (and are most relevant for this course) include copying from another student's work or allowing another student to copy from your own work; plagiarism; revising and resubmitting your work for regrading without the instructor's knowledge and consent; representing as one's own work the work of another. See http://communitystandards.stanford.edu/ for more information on the Honor Code.