Math 214: Quiz 4

April 5, 2012

Instructions. You have 20 minutes for this quiz. Please work in silence, and use additional sheets for your work. No calculators, phones, computers, books, et cetera are allowed during the quiz. Make sure that your solution is easy to read, and that the final answer is clearly visible.

1. Consider the matrix
$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 9999 \\ 1 & 9999 & 2 & 0 & 2 \\ 9999 & 0 & 1 & 1 & 2 \\ 1 & -2 & 9999 & 1 & 0 \\ 1 & -1 & 0 & 9999 & 2 \end{pmatrix}.$$
Is $\det(A)$ positive or negative or zero? Explain briefly.

2. Find the characteristic polynomial of
$$A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & 4 \\ 1 & 0 & -1 \end{pmatrix}.$$

3. Let A be a (2×2)-matrix with characteristic polynomial $\lambda^2 - 2\lambda + 4$. Calculate the characteristic polynomial for
$$\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} + 2 \cdot A.$$

4. Let V be a plane in $\mathbf{R}^3$. Let $f : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear transformation defined as follows: $f(v)$ is the result of rotating by $\pi/2$ the orthogonal projection of v onto V. Calculate all real eigenvalues of f with algebraic multiplicities.

5. Are the following true or false?

   (a) Any invertible matrix A commutes with its adjoint $\mathrm{adj}(A)$, i.e., one has $A \cdot \mathrm{adj}(A) = \mathrm{adj}(A) \cdot A$.

   (b) If a (4×4)-matrix A has one real eigenvalue with algebraic multiplicity 1, then it must have at least one other different real eigenvalue.

   (c) If a (5×5)-matrix A has one real eigenvalue with algebraic multiplicity 1, then it must have at least one other different real eigenvalue.

   (d) If an $(n \times n)$-matrix A has no real eigenvalues, nor does A^2.