Math 214: Quiz 3

March 15, 2012

Instructions. You have 20 minutes for this quiz. Please work in silence, and use additional sheets for your work. No calculators, phones, computers, books, et cetera are allowed during the quiz. Make sure that your solution is easy to read, and that the final answer is clearly visible.

1. Let V be the subspace of $\mathbf{R}^3$ defined by $x = 2y + 3z$. Find an orthonormal basis for V.
2. Calculate the QR decomposition of $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 4 \\ 1 & -2 & -1 \end{pmatrix}$
3. Calculate $\dim(\ker(A^T A))$ for $A = \begin{pmatrix} 2 & 3 & 5 \\ 0 & 7 & 3 \\ 0 & 0 & 2 \end{pmatrix}$
4. Are the following true or false?
   (a) There is a (2×2)-matrix A with $\ker(A) = \mathrm{im}(A)$.
   (b) There is a (3×3)-matrix A with $\ker(A) = \mathrm{im}(A)$.
   (c) If A is a (4×3)-matrix and B is a (3×4)-matrix, then the matrix $A \cdot B$ is not orthogonal.
   (d) There are only finitely many orthogonal (4×4)-matrices whose entries are integers.
   (e) If A is a square matrix, and $\|Ax\| = 1$ for all unit vectors x, then A is orthogonal.
   (f) If A is a square matrix, then A is orthogonal if and only if A^T is so.