Math 214: Quiz 2

February 14, 2012

Instructions. You have 20 minutes for this quiz. Please work in silence, and use additional sheets for your work. No calculators, phones, computers, books, etc. are allowed during the quiz. Make sure that your solution is easy to read, and that the final answer is clearly visible.

1. Determine if the following matrix has an inverse; if it does, then calculate the inverse.

$$\begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

2. Let $f : \mathbf{R}^3 \to \mathbf{R}^2$ be the linear transformation defined by

$$f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+y \\ y+z \end{pmatrix}$$

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   (a) Write down a (2×3)-matrix A such that $f(v) = A \cdot v$ for any vector v in $\mathbf{R}^3$.

   (b) Calculate $\dim(\ker(f))$.

3. Find all (2×2)-matrices A that commute with $B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, i.e., that satisfy $A \cdot B = B \cdot A$.

4. Write down two different bases for $\mathbf{R}^2$ without repeating vectors.

5. Are the following true or false?

   (a) A rotation in $\mathbf{R}^2$ has no kernel.

   (b) If A is an $(n \times m)$-matrix with rank n, then $\dim(\ker(A)) = 0$.

   (c) A reflection in $\mathbf{R}^2$ can be written as a diagonal matrix with respect to some choice of basis.

   (d) A rotation in $\mathbf{R}^2$ can be written as a diagonal matrix with respect to some choice of basis.