Math 214: Quiz 1

January 26, 2012

Instructions. You have 20 minutes for this quiz. Please work in silence, and use additional sheets for your work. No calculators, phones, computers, books, etc. are allowed during the quiz.

1. Calculate the row reduced echelon form of

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$$

2. Find a (2×2)-matrix A satisfying:

$$A \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad A \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

3. Find all (2×2)-matrices A satisfying

$$A \cdot \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

4. Are the following true or false?

   (a) The map $f : \mathbf{R}^2 \to \mathbf{R}^2$ given by

   $$f \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+1 \\ y \end{pmatrix}$$

   is a *linear* transformation.

   (b) If a (2×2)-matrix A represents a horizontal shear, then A preserves lengths, i.e., for any vector x in $\mathbf{R}^2$, we have

   $$\left(A \cdot x\right) \cdot \left(A \cdot x\right) = x \cdot x$$

   (c) Fix a line L in $\mathbb{R}^2$ that goes through the origin. Let A be the matrix that represents orthogonal projection onto L, and let B be the matrix that represents reflection about L. Then $A \cdot B = A$.

   (d) If A is a scaling matrix, then there exists a matrix B such that $B^2 = A$.

   (e) If A is a rotation matrix, then there exists a matrix B such that $B^2 = A$.

   (f) If A is an orthogonal projection matrix, then there exists a matrix B such that $B^2 = A$.