Solutions to the True/False questions for the midterm

(a) If a (2×2)-matrix A has determinant 5, then A can not be a matrix for othogonal projection onto a line L. **True**

(b) Let $f : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation. If there exists a basis $\mathcal{B}$ of $\mathbf{R}^2$ such that the $\mathcal{B}$-matrix of f is $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}_{\mathcal{B}}$, then $f(x) = -x$ for all vectors x. **True**

(c) If u, v, and w are non-zero vectors in $\mathbf{R}^2$, then w is a linear combination of u and v. **False**

(d) Let V be a subspace of $\mathbf{R}^n$. If the orthogonal projection map $\mathrm{proj}_V : \mathbf{R}^n \to \mathbf{R}^n$ is invertible, then $V = \mathbf{R}^n$. **True**

(e) There is a (2×3)-matrix A and a (3×2)-matrix B such that $A \cdot B$ is the identity matrix. **True**

(f) There is a (2×3)-matrix A and a (3×2)-matrix B such that $B \cdot A$ is the identity matrix. **False**

(g) Let A, B, and C be $(n \times n)$-matrices. If A is similar to B, and B is similar to C, then A is similar to C. **True**

(h) Every matrix is similar to a diagonal matrix. **False**

(i) Let V be a subspace of $\mathbf{R}^n$. Then there is an $(n \times n)$-matrix A with $\ker(A) = V$. **True**

(j) Let V be a subspace of $\mathbf{R}^n$. Then there is an $(n \times n)$-matrix A with $\mathrm{im}(A) = V$. **True**