Math 214 — Midterm 2

Blue Version Answers

by Michael Cap Khoury

Problem 1:

The determinant is 9. This is best done by Laplace expansion or swapping the second and third rows to get a block diagonal matrix.

Problem 2:

A basis of this space is $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$. Starting from the matrix $A = \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 0 & -1 \end{pmatrix}$, the projection matrix is given by $A(A^TA)^{-1}A^T = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$.

Alternatively, we could use Gram-Schmidt and then use the simpler formula QQ^T.

Problem 3:

a) $\operatorname{adj} A = \begin{pmatrix} 9 & 3 & -11 \\ 7 & 2 & -8 \\ -6 & -2 & 7 \end{pmatrix}$

b) $A^{-1} = \begin{pmatrix} -9 & -3 & 11 \\ -7 & -2 & 8 \\ 6 & 2 & -7 \end{pmatrix}$

Problem 4:

The original system is $A\vec{x} = \vec{b}$, for $A = \begin{pmatrix} 2 & 1 \\ 1 & -1 \\ 1 & 0 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix}$. The normal equation $A^TA\vec{x} = A^T\vec{b}$ is $\begin{pmatrix} 5 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 16 \\ 2 \end{pmatrix}$. The least squares solution is $x = 10/3, y = -2/3$.

Problem 5:

$$\begin{pmatrix} 3/13 & 4/13 & 12/13 \\ 4/13 & -12/13 & 3/13 \\ 12/13 & 3/13 & -4/13 \end{pmatrix}$$

Problem 6:

a) $Q = \begin{pmatrix} 2/5 & 1/2 \\ 2/5 & -1/2 \\ 2/5 & 1/2 \\ 2/5 & -1/2 \\ 3/5 & 0 \end{pmatrix}, R = \begin{pmatrix} 5 & 10 \\ 0 & 6 \end{pmatrix}$

b) $\begin{pmatrix} 2/5 \\ 2/5 \\ 2/5 \\ 2/5 \\ 3/5 \end{pmatrix}, \begin{pmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \\ 0 \end{pmatrix}$

c) $S_{\mathcal{A}\to\mathcal{B}} = R = \begin{pmatrix} 5 & 10 \\ 0 & 6 \end{pmatrix}$. $[\vec{v}]_{\mathcal{B}} = \begin{pmatrix} 40 \\ 18 \end{pmatrix}, [\vec{w}]_{\mathcal{B}} = \begin{pmatrix} -5 \\ -6 \end{pmatrix}$

d) $\cos\theta = \frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\,\|\vec{w}\|} = \frac{-308}{\sqrt{1924}\sqrt{51}}$ (this makes the angle between them about 154°)

Problem 7:

a) $\begin{pmatrix} 3 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

b) $\ker T$ is spanned by $\{1\}$

c) $\operatorname{im} T$ is spanned by $\{3t^2+2t, t\}$ (or just by $\{t^2, t\}$)

d) 1

e) 2

f) 0 (it is not invertible)

Problem 8:

a) True b) False c) False d) True e) False f) True g) True h) False i) False j) True