Math 214 — Midterm 1
(Blue Version)

This is an 80-minute exam, but you have the full 110-minute class period to complete it. There are a total of 100 points possible; the value of each problem is marked. Partial credit may be given for significant progress toward the solution to a problem. Except where otherwise indicated, you must show sufficient work to make it clear to me what you are doing and why in order to obtain full credit.

Problem 1 (10 pts)

The Mysterious Merchant sells three types of tea. Dragon tea costs 2 gold coins for a sack with 25 pounds of tea. Serenity Tea costs 3 gold coins for a sack with 20 pounds of tea. Miracle Tea costs 5 gold coins for a sack with 30 pouns of tea. If Datura spends 25 gold coins to buy nine sacks, containing a total of pounds of tea, how many sacks of each type did Datura buy?

Problem 2 (10 pts)

Compute the orthogonal projection of the vector $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ onto the line L spanned by $\begin{pmatrix} 3 \\ 4 \\ 12 \end{pmatrix}$.

Problem 3 (10 pts)

Write down a 4×5 matrix so that the image of your matrix (that is, the image of the transformation it represents) is the space spanned by $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$.

Problem 4 (10 pts)

Write down the matrix which represents the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ satisfying

$$T\begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 21 \\ 15 \\ -12 \end{pmatrix}, \qquad T\begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 10 \\ 12 \\ 14 \end{pmatrix}.$$

Problem 5 (10 pts)

Write down three different bases of $\mathbb{R}^3$. You may not repeat any vectors in your solution. (That is, your answer must involve nine different vectors.)

Problem 6 (15 pts)

Consider the matrix

$$A = \begin{pmatrix} 1 & 4 & 2 & 1 & 1 \\ 0 & 1 & 1 & -1 & 0 \\ 1 & 5 & 3 & 0 & 0 \end{pmatrix}$$

and let T be the transformation represented by A.

a) Write down a basis for the kernel of T.

b) Write down a basis for the image of T.

c) What is the rank of A?

d) What is the nullity of A?

Problem 7 (15 pts)

Consider the following matrices:

$$A = \begin{pmatrix} 1 & 4 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -5 & 2 & 1 \end{pmatrix}$$

a) Compute $C = AB$.

b) Compute A^{-1} or explain why A is not invertible.

c) Compute B^{-1} or explain why B is not invertible.

d) Compute C^{-1} or explain why C is not invertible. *(Hint: there is more than one way to do this part.)*

Problem 8 (20 pts)

Indicate whether each item is true or false. Each item is worth 2 points. No explanation necessary, no partial credit possible.

a) **T / F ?** Every system of five linear equations in three variables is consistent.

b) **T / F ?** For any 5×6 matrix A and any vector $\vec{b}$, the system $A\vec{x} = \vec{b}$ does not have a unique solution.

c) **T / F ?** If A is a 4×4 matrix and rref(A) is not the identity matrix, then the columns of A must be linearly independent.

d) **T / F ?** The map from the plane to itself which sends a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} x+1 \\ y+1 \end{pmatrix}$ is linear.

e) **T / F ?** The matrix $\begin{pmatrix} 3/5 & 4/5 \\ -4/5 & 3/5 \end{pmatrix}$ represents a rotation.

f) **T / F ?** If V is a five-dimensional space and $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5$ span V, then $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5$ are linearly independent.

g) **T / F ?** If A is an invertible matrix, then A is square.

h) **T / F ?** If the list of vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{e}, \vec{f}, \vec{g}$ contains three redundant vectors, then the list of vectors $\vec{g}, \vec{f}, \vec{e}, \vec{d}, \vec{c}, \vec{b}, \vec{a}$ also contains at least three redundant vectors.

i) **T / F ?** If a 3×7 matrix represents a transformation T and the image of T is a plane, the kernel of T is a line.

j) **T / F ?** If L is a 1-dimesnionsl subspace of $\mathbb{R}^3$, then there are infinitely many subspaces of $\mathbb{R}^3$ containing L.