The Work of Leslie Valiant

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ABSTRACT

On Saturday, May 30, one day before the start of the regular STOC 2009 program, a workshop was held in celebration of Leslie Valiant's 60th birthday. Talks were given by Jin-Yi Cai, Stephen Cook, Vitaly Feldman, Mark Jerrum, Michael Kearns, Mike Paterson, Michael Rabin, Rocco Servedio, Paul Valiant, Vijay Vazirani, and Avi Wigderson. The workshop was organized by Michael Kearns, Rocco Servedio, and Salil Vadhan, with support from the STOC local arrangements team and program committee.

To accompany the workshop, here we briefly survey Valiant's many fundamental contributions to the theory of computing.

Categories and Subject Descriptors

A.0 [**General Literature**]: Biographies/autobiographies; F.0 [**Theory of Computation**]: General

General Terms

Theory

1. INTRODUCTION

Les Valiant singlehandedly created, or completely transformed, several fundamental research areas of computer science. Here we list some of those areas, and some of Valiant's seminal contributions to each.

2. COMPUTATIONAL LEARNING THEORY

Valiant's "Theory of the Learnable" [13] created the area of Computational Learning Theory. Mainstream research in Machine Learning today has embraced Valiant's viewpoint as a path to understand and design intelligent systems. Philosophically, Valiant's work presents a computational theory that mirrors or at least serves as a metaphor for one of the most basic human activities: learning. Pragmatically, this work suggests a framework for the development of algorithms that adapt their behavior in response to feedback from the environment. Needless to say, attempts to model the learning process were made before Valiant. His contribution was in providing a general framework, as well as very concrete computational models for studying questions relating to the learning process, namely the PAC Learning Model and its variants. This led to well-defined research directions and created a language that enabled this study to mature into a real scientific theory, with techniques, results and connections to the rest of computer science. The resulting theory turned out to have numerous applications in a very wide area of computer practice (e.g., in Computer Vision and Natural Language Processing) as well as inform other scientific disciplines (e.g., the study of proteins and the cognitive functions of the brain). It is impossible to give details here of Valiant's numerous concrete contributions to computational learning theory. We only mention a recent work of his that attempts to present a computational theory of evolution, which builds on his learning framework [20].

3. COMPUTATIONAL COMPLEXITY

Valiant's seminal contributions to this field are numerous, and we will touch on only a few of them. We start with his definition of the class #P of counting problems [8], and his proof that the Permanent is complete for this class [7]. Beyond the direct interest in this class, these innovations have served computational complexity in unforeseen ways. For example, the class #P was shown to contain the polynomial time hierarchy [3]. Moreover, the special properties of Permanent, such as random-self-reducibility, played a key role in unveiling the power of interactive proofs, PCPs, program checking and more. Next, Valiant's algebraic analogue of Boolean complexity (for the computation of polynomials) [6] served as the pivot for much subsequent research. Valiant's own seminal contributions to this research include identifying the Permanent and Determinant as complete problems for two natural classes of algebraic computations [6], proving that (in this model) negation can be exponentially more powerful than monotone computation [9], and last but not least, surprisingly proving that sequential computation can always be parallelized (in contrast to what we believe to be true in Boolean complexity) [21]. Valiant has also made numerous contributions to the study of Boolean circuit complexity. In particular, the fundamental concepts of superconcentrators (see below) and rigidity have originated from his works [4, 5], and still serve as some of the very few tools for proving (weak, so far) time-space trade-offs and circuit lower bounds. We mention that superconcentrators were also the inspiration (in a precise technical sense) for the first construction of error-correcting codes that are optimal to within constant factors in all aspects: linear distance, constant rate, and linear-time encoding and decoding [2]. Valiant's contributions have also had a significant impact on other areas of complexity theory. The list includes

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his polynomial-size monotone formula for majority [12], the study of the complexity of finding unique solutions [22], and the relation between approximate counting and uniform generation [1].

4. PARALLEL AND DISTRIBUTED COMPUTATION

Perhaps the most striking single contribution to this area is Valiant's randomized distributed routing algorithm (and its beautiful analysis) [11]. Randomized routing provides a way of avoiding congestion in sparse networks, something unavoidable for any deterministic algorithm. This algorithm and its follow-ups have become basic tools in parallel and distributed architectures, memory management and load balancing. In a survey on parallel computing in the early 1980s [10], Valiant posed several challenges that shaped the direction of this field. These challenges called for attempting to parallelize some seemingly "inherently sequential" problems like "maximal independent set" and "maximum matching", and addressing them gave rise to the development of some of the most basic algorithmic techniques we have in this field. Another such basic technique is presented in Valiant's parallelization of polynomial computation (mentioned above) [21]. In addition, the aforementioned "superconcentrators", which Valiant explicitly and optimally constructed in his work on circuit complexity [4], serve as fundamental networks, which in turn underlie distributed sorting, searching and routing algorithms. Finally, Valiant's Bulk-Synchronous Parallel (BSP) Model bridges the gap between software and hardware for parallel computation, enabling arbitrary parallel algorithms to be efficiently compiled onto a wide variety of parallel architectures [14].

5. COGNITIVE THEORIES

Valiant has been fascinated with the workings of the brain. Following his work on computational learning theory, he tried to explain how the (assumed) architecture of the brain can support complex cognitive functions that humans seem to perform trivially, like memory, complex pattern recognition, natural language recognition, rule learning and more. His book "Circuits of the Mind" [15] offers a concrete mathematical model of the brain, together with programs that (under this architecture) actually perform such cognitive functions. Valiant has continued this research further in a body of subsequent work that elaborates his theory, relates it to ongoing discoveries in neuroscience, and provides experimental support for the theory (e.g. [17, 18]).

6. HOLOGRAPHIC ALGORITHMS

In a recent set of papers (e.g. [19]), Valiant suggests a novel algorithmic technique, which is able to solve counting problems and constraint satisfaction problems that appear to be inaccessible by any previous method. Abstractly, these algorithms provide reductions to a problem in P, specifically the problem of counting the number of perfect matchings in planar graphs. However, the reductions are "holographic", namely they are many-to-many (as opposed to the standard many-to-one reductions which are commonly used), and seem to use cancellation (i.e., negation) in a mysterious way. If this reminds you of quantum computation, it should! Valiant showed that his model may be viewed as a submodel of the standard quantum Turing machine that has a classical simulation in polynomial time [16]. The ramifications of these ideas, both as a new classical algorithmic technique as well as a quantum computational model are currently being pursued by a number of researchers (as was the "fate" of many of Valiant's earlier works).

7. SUMMARY

Valiant has made enormous contributions to computer science; his work has had a huge impact on both its theory and practice, as well as the embodiment of computational principles in other scientific disciplines. We all became addicted to this remarkable throughput, and expect more. Happy birthday Les!

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