Subspace arrangements, graph rigidity and derandomization through submodular optimization

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Abstract

This paper presents a deterministic, strongly polynomial time algorithm for computing the matrix rank for a class of symbolic matrices (whose entries are polynomials over a field). This class was introduced, in a different language, by Lovász [Lov] in his study of flats in matroids, and proved a duality theorem putting this problem in $NP \cap coNP$. As such, our result is another demonstration where "good characterization" in the sense of Edmonds leads to an efficient algorithm. In a different paper Lovász [Lov79] proved that all such symbolic rank problems have efficient probabilistic algorithms, namely are in BPP. As such, our algorithm may be interpreted as a derandomization result, in the long sequence special cases of the PIT (Polynomial Identity Testing) problem. Finally, Lovász and Yemini [LoYe] showed how the same problem generalizes the graph rigidity problem in two dimensions. As such, our algorithm may be seen as a generalization of the well-known deterministic algorithm for the latter problem.

There are two somewhat unusual technical features in this paper. The first is the translation of Lovász' flats problem into a symbolic rank one. The second is the use of submodular optimization for derandomization. We hope that the tools developed for both will be useful for related problems, in particular for better understanding of graph rigidity in higher dimensions.