

# Expanders that beat the eigenvalue bound: explicit construction and applications

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## Abstract

For every  $n$  and  $0 < \delta < 1$ , we construct graphs on  $n$  nodes such that every two sets of size  $n^\delta$  share an edge, having essentially optimal maximum degree  $n^{1-\delta+o(1)}$ . Using known and new reductions from these graphs, we derive new explicit constructions of:

1. A  $k$  round sorting algorithm using  $n^{1+1/k+o(1)}$  comparisons.
2. A  $k$  round selection algorithm using  $n^{1+1/(2^k-1)+o(1)}$  comparisons.
3. A depth 2 superconcentrator of size  $n^{1+o(1)}$ .
4. A depth  $k$  wide-sense nonblocking generalized connector of size  $n^{1+1/k+o(1)}$ .

All of these results improve on previous constructions by factors of  $\Omega(1)$ , and are optimal to within factors of  $o(1)$ . These results are based on an improvement to the extractor construction of Nisan & Zuckerman: our algorithm extracts an asymptotically optimal number of random bits from a defective random source using a small additional number of truly random bits.