One-way multi-party communication lower bound for pointer jumping with applications

Emanuele Viola*

Avi Wigderson†

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Abstract

In this paper we study the one-way multi-party communication model, in which every party speaks exactly once in its turn. For every fixed k, we prove a tight lower bound of $\Omega\left(n^{1/(k-1)}\right)$ on the probabilistic communication complexity of pointer jumping in a k-layered tree, where the pointers of the i-th layer reside on the forehead of the i-th party to speak. The lower bound remains nontrivial even for $k = (\log n)^{1/2-\Omega(1)}$ parties. Previous to our work a lower bound was known only for $k = 3$ [BHK], and in very restricted models for $k > 3$ [DJS, Cha]. Our results have the following consequences to other models and problems, extending previous work in several directions.

The one-way model is strong enough to capture *general* (non one-way) multi-party protocols of bounded rounds. Thus we generalize to this multi-party model results on two directions studied in the classical 2-party model (e.g. [PS, NW]). The first is a round hierarchy: We give an exponential separation between the power of r and $2r$ rounds in general probabilistic k-party protocols, for any fixed k and r. The second is the relative power of determinism and nondeterminism: We prove an exponential separation between nondeterministic and deterministic communication complexity for general k-party protocols with r rounds, for any fixed k, r.

The pointer jumping function is weak enough to be a special case of the well-studied disjointness function. Thus we obtain a lower bound of $\Omega\left(n^{1/(k-1)}\right)$ on the probabilistic complexity of k-set disjointness in the one-way model, which was known only for $k = 3$ parties. Our result also extends a similar lower bound for the weaker simultaneous model, in which parties simultaneously send one message to a referee [BPSW].

Finally, we infer an exponential separation between the power of different orders in which parties send messages in the one-way model, for every fixed k. Previous to our work such a separation was only known for $k = 3$ [NW].

Our lower bound technique, which handles functions of high discrepancy, may be of independent interest. It provides a "party-elimination" induction, based on a restricted form of a direct-product result, specific to the pointer jumping function.

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