

Depth-3 Arithmetic Formulae over Fields of Characteristic Zero

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Abstract

In this paper we prove quadratic lower bounds for depth-3 arithmetic circuits over fields of characteristic zero. Such bounds are obtained for the elementary symmetric functions, the (trace of) iterated matrix multiplication, and the determinant. As corollaries we get the first nontrivial lower bounds for computing polynomials of constant degree, and a gap between the power of depth-3 arithmetic circuits and depth-4 arithmetic circuits. We also give new shorter formulae of constant depth for the elementary symmetric functions.

The main technical contribution relates the complexity of computing a polynomial in this model to the wealth of partial derivatives it has on every affine subspace of small co-dimension. Lower bounds for related models utilize an algebraic analog of Nechiporuk lower bound on Boolean formulae.