# Read-Once Branching Programs, Rectangular Proofs of the Pigeonhole Principle and the Transversal Calculus

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**Abstract**
We investigate read-once branching programs for the following search problem: given a Boolean $m x n$ matrix with $m>n$, find either an all-zero row, or two 1's in some column. Our primary motivation is that this model's regular resolution proofs of the pigeonhole principle $PHP^m_n$, and that for $m>n^2$ no lower bounds are known for the length of such proofs. We prove exponential lower bounds (for arbitrary large $m!$) if we further restrict this model by requiring the branching program *either* to finish one row of queries before asking queries about another row (the *row model* ) or put the dual column restriction (the *column model* ).

Then we investigate a special class of resolution proofs for $PHP^m_n$ that operate with positive clauses of rectangular shape; we call this fragment the *rectangular calculus.* We show that all known *upper* bounds on the size of resolution proofs of $PHP^m_n$ actually give rise to proofs in this calculus and, inspired by this fact, also give a remarkably simple "rectangular" reformulation of the Haken-Buss-Turán lower bound for the case $m << n^2$. Finally we show that the rectangular calculus is equivalent to the column model on the one hand, and to *transversal calculus* on the other hand, where the latter is a natural proof system for estimating from below the transversal size of set families. In particular, our exponential lower bound for the column model translates both to the rectangular and transversal calculi.