

Entropy Waves, the Zig-Zag Graph Product, and New Constant-Degree Expanders and Extractors

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Abstract

The main contribution of this work is a new type of graph product, which we call the **zig-zag product**. Taking a product of a large graph with a small graph, the resulting graph inherits (roughly) its size from the large one, its degree from the small one, and its expansion properties from both! Iteration yields simple explicit constructions of constant-degree expanders of every size, starting from one constant-size expander.

Critical to our intuition (and simple analysis) of the properties of this graph product is the view of expanders as functions which act as “entropy wave” propagators – they transform probability distributions in which entropy is concentrated in one area to distributions where that concentration is dissipated. In these terms, the graph product affords the constructive interference of two such waves.

A variant of this product can be applied to extractors, giving the first explicit extractors whose seed length depends (poly) logarithmically on only the entropy deficiency of the source (rather than its length) and that extract almost all the entropy of high min-entropy sources. These high min-entropy extractors have several interesting applications, including the first constant-degree explicit expanders which bear the “eigenvalue bound.”