ABSTRACT

We assume that for some *fixed* large enough integer d, the symmetric group S_d can be generated as an expander using $d_{1=30}$ generators. Under this assumption, we explicitly construct an infinite family of groups G_n, and explicit sets of generators Y_n _ G_n, such that all generating sets have bounded size (at most $d_{1=7}$), and the associated Cayley graphs are all expanders.
The groups G_n above are very simple, and completely different from previous known examples of expanding groups. Indeed, G_n is (essentially) all symmetries of the d-regular tree of depth n.
The proof is completely elementary, using only simple combinatorics and linear algebra. The recursive structure of the groups G_n (iterated wreath products of the alternating group A_d) allows for an inductive proof of expansion, using the group theoretic analogue [4] of the zig-zag graph product of [37]. The explicit construction of the generating sets Y_n uses an efficient algorithm for solving certain equations over these groups, which relies on the work of [32] on the commutator width of perfect groups.
We stress that our assumption above on *weak* expansion in the symmetric group is an open problem. We conjecture that it holds for all d. We discuss known results related to its likelihood in the paper.