Quadratic Dynamical Systems

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Abstract

The main purpose of this paper is to promote the study of computational aspects, primarily the convergence rate, of nonlinear dynamical systems from a combinatorial perspective.

We identify the class of symmetric quadratic systems. Such systems have been widely used to model phenomena in the natural sciences, and also provide an appropriate framework for the study of genetic algorithms in combinatorial optimization. We prove several fundamental general properties of these systems, including a characterization of the set of fixed points to which the system converges.

We go on to give a detailed analysis of a quadratic system defined in a natural way on probability distributions over the set of matchings in a graph. In particular, we prove that convergence to the limit requires only polynomial time when the graph is a tree. This result demonstrates that such systems, though nonlinear, are amenable to quantitative analysis.