# Direct Product Results and the GCD Problem, in Old and New Communication Models

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**Abstract**
This paper contains several results regarding the communication
complexity model and the 2-prover games model, which are based on interaction between the two models:

1. We show how to improve the rate of exponential decrease in the parallel repetition theorem of [Ra] in terms of the communication complexity of the verifier's predicate.

2. We apply the improved parallel repetition theorem of 2-prover games to derive, for the first time, a direct product theorem for communication complexity.

The second derivation uses a common generalization of the two models, which is independently interesting. We initiate a study of its power by considering the $GCD$ problem, and some variations of it, which exhibit a power gap between the new model and the classical communication complexity model. This gap is partly based on the following upper bounds: Given $n$-bit inputs $x$ and $y$ to Alice and Bob respectively, they can achieve the tasks below with very high probability using only $O(n= log n)$ communication bits:

1. Decide if $GCD(x; y) = 1$ and $b$ (by Bob), satisfying $a _ x + b _ y = 1$.

Observe that the outputs in the second task are in general of length $(n)$. A complete analysis of the communication complexity of these two problems (in several models and modes) is given.