Characterizing Non-Deterministic Circuit Size
(*Four variations on one theme*)

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Abstract

Consider the following simple communication problem. Fix a universe $*U*$ and a family Ω of subsets of $*U*$. Players I and II receive, respectively, an element $\alpha ∈ *U*$ and a subset $*A* ∈ \Omega$. Their task is to find a subset $*B*$ of $*U*$ such that $|*A*∩*B*|$ is even and $\alpha ∈ *B*$. With every Boolean function $*f*$ we associate a collection Ω_f of subsets of *U* = *f^{-1}(0)$*, and prove that its (one round) communication complexity completely determines the *size* of the smallest nondeterministic circuit for $*f*$.

We propose a linear algebraic variant to the general approximation method of Razborov, which has exponentially smaller description. We use it to derive four different combinatorial problems (like the one above) that characterize $*NP*$. These are tight, in the sense that they can be used to prove super-linear circuit size lower bounds. Combined with Razborov's method, they present a purely combinatorial framework in which to study the $*P* vs. *NP* vs. *co – NP*$ question.