Characterizing Non-Deterministic Circuit Size
(*Four variations on one theme*)

Mauricio Karchmer
Avi Wigderson

Abstract

Consider the following simple communication problem. Fix a universe $U$ and a family $\Omega$ of subsets of $U$. Players I and II receive, respectively, an element $\alpha \in U$ and a subset $A \in \Omega$. Their task is to find a subset $B$ of $U$ such that $|A \cap B|$ is even and $\alpha \in B$. With every Boolean function $f$ we associate a collection $\Omega_f$ of subsets of $U = f^{-1}(0)$, and prove that its (one round) communication complexity completely determines the size of the smallest nondeterministic circuit for $f$.

We propose a linear algebraic variant to the general approximation method of Razborov, which has exponentially smaller description. We use it to derive four different combinatorial problems (like the one above) that characterize $NP$. These are tight, in the sense that they can be used to prove super-linear circuit size lower bounds. Combined with Razborov’s method, they present a purely combinatorial framework in which to study the $P$ vs. $NP$ vs. $co-NP$ question.