Monotone Circuits for Connectivity Require Super-Logarithmic Depth

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Abstract

It is proved here that every monotone circuit which tests st-connectivity of an undirected graph on n nodes has depth $\Omega(log^2 n)$. This implies a superpolynomial $(n^{\Omega(log n)})$ lower bound on the size of any monotone formula for st-connectivity.

The proof draws intuition from a new characterization of circuit depth in terms of communication complexity. Within the same framework, a very simple and intuitive proof is given of a depth analogue of a theorem of Khrapchenko concerning formula size lower bounds.