

The Complexity of Parallel Search

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Abstract

This paper studies parallel search algorithms within the framework of independence systems. It is motivated by earlier work on parallel algorithms for concrete problems such as the determination of a maximal independent set of vertices or a maximum matching in a graph and by the general question of determining the parallel complexity of a search problem when an oracle is available to solve the associated decision problem. Our results provide a parallel analog of the self-reducibility process that is so useful in sequential computation. An abstract independence system is specified by a ground set E and a family of subsets of E called the independent sets; it is required that every subset of an independent set be independent. We investigate parallel algorithms for determining a maximal independent set through oracle queries of the form “Is the set A independent?”, as well as parallel algorithms for determining a maximum independent set through queries to a more powerful oracle called a rank oracle. We also study these problems for three special types of independence systems: matroids, graphic matroids, and partition matroids. We derive lower and upper bounds on the deterministic and randomized complexity of these problems. These bounds are sharp enough to give a clear picture of the processor-time trade-offs that are possible, to establish that randomized parallel algorithms are much more powerful than deterministic ones, and to show that even randomized algorithms cannot make effective use of extremely large numbers of processors.