

# Non-deterministic Communication Complexity with Few Witnesses

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## **Abstract**

We study non-deterministic communication protocols in which no input has too many witnesses. Define  $n_k(f)$  to be the maximum complexity of a non-deterministic protocol for the function  $f$  in which each input has at most  $k$  witnesses. We present two different lower bounds for  $n_k(f)$ . Our first result shows that  $n_k(f)$  is bounded below by  $\Omega(\sqrt{c(f)/k})$  where  $c(f)$  is the deterministic complexity. Our second result bounds  $n_k(f)$  by  $\log(\text{rk}(M_f)/k-1)$  where  $\text{rk}(M_f)$  is the rank of the representing matrix of  $f$ . As a consequence, it follows that the communication complexity analogue of the Turing-complexity class  $\text{Few P}$  is equal to the analogue of the class  $\text{P}$ .