Randomized vs. Deterministic Decision Tree Complexity for Read-Once Boolean Functions

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Abstract

We consider the deterministic and the randomized decision tree complexities for Boolean functions, denoted $DC(f)$ and $RC(f)$, respectively. A major open problem is how small $RC(f)$ can be with respect to $DC(f)$. It is well known that $RC(f) \geq DC(f)^0.5$ for every Boolean function f (called “0.5-exponent"). On the other hand, some Boolean function f is known to have $RC(f) = \Theta(DC(f)^{0:753:::})$ (or “0.753-exponent"). It is not known whether there is a Boolean function with exponent smaller than 0.753... Likewise, no lower bound for arbitrary Boolean functions with exponent greater than 0.5 is known.

Our result is a 0.51 lower bound on the exponent for every read-once function. Read-once means that each input variable appears exactly once in the Boolean formula representing the function. To obtain this result we generalize an existing lower bound technique and combine it with restriction arguments. This result provides a lower bound of $n^0.51$ on the number of positions that have to be evaluated by any randomized $\alpha - \beta$ pruning algorithm computing the value of any two-person zero-sum game tree with n final positions.