On Read-Once Threshold Formulae and their Randomized Decision Tree Complexity

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Abstract

$TC^0$ is the class of functions computable by polynomial-size, constant-depth formulae with threshold gates. Read-Once $R0-TC^0$ is the subclass of $TC^0$ which restricts every variable to occur exactly once in the formula.

Our main result is a (tight) linear lower bound on the randomized decision tree complexity of any function in $R0-TC^0$.

This relationship between threshold circuits and decision trees bears significance on both models of computation. Regarding decision trees, this is the first class of functions for which such a strong bound is known. Regarding threshold circuits, it may be considered as a possible first step towards proving $TC^0 \not= NC^1$; generalizing our lower bounds to all functions in $TC^0$ will establish this separation.

Another structural result we obtain is that a read-once threshold formula uniquely represents the function it computes.