Boolean Complexity Classes vs. their Arithmetic Analogs

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Abstract
This paper provides logspace and small circuit depth analogs of the result of Valiant and Vazirani, which is a randomized (or nonuniform) reduction from \(\text{NP}\) to its arithmetic analog \(\text{PLUSMINUS P}\). We show a similar randomized reduction between the Boolean classes \(\text{NL}\) and semiunbounded fan-in Boolean circuits and their arithmetic counterparts. These reductions are based on the Isolation Lemma of Mulmuley, Vazirani and Vazirani. Combinatorically our results can be viewed as simple (logspace) transformations of existential quantifiers into counting quantifiers in graphs and shallow circuits.