Computing graph properties by randomized subcube partitions

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Abstract
We prove a new lower bound on the randomized decision tree complexity of monotone graph properties. For a monotone graph property A of graphs on n vertices, let $p=p(A)$ denote the threshold probability of A, namely the value of p for which a random graph from $G(n,p)$ has property A with probability $1/2$. Then the expected number of queries made by any decision tree for A on such a random graph is at least $\omega(n^2/max{pn, log n})$.

Our lower bound holds in the subcube partition model, which generalizes the decision tree model. The proof combines a simple combinatorial lemma on subcube partitions (which may be of independent interest) with simple graph packing arguments. Our approach motivates the study of packing of ``typical'' graphs, which may yield better lower bounds.