Superconcentrators, Generalizers, and Generalized Connectors with Limited Depth

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Abstract

We show that the minimum possible size of an n-superconcentrator with depth $2k \geq 4$ is $\Theta(n)(k, n))$, where $\Lambda(k, .)$ is the inverse of a certain function at the k-th level of the primitive recursive hierarchy. It follows that the minimum possible depth of an n-superconcentrator with linear size is $\Theta(\Beta(n))$, where $\Beta$ is the inverse of a function growing more rapidly than any primitive recursive function. Similar results hold for generalizers.

We give a simple explicit construction for a $(d_1 \ldots d_k)$-generalizer with depth k and size $(d_1 + \ldots +d_k)d_1 \ldots d_k$. This is applied to give a simple explicit construction for a generalized n-connector with depth $2k-3$ and size $(2d_1 +3d_2 + \ldots +3d_{k-1}+2d_k) d_1 \ldots d_k$. These are the best explicit constructions currently available. We also show that, for each fixed $k \geq 2$, the minimum possible size of a generalized n-connector with depth k is $\Omega(n^{1+1/k})$ and $O((n log n)^{1+1/k})$.