Short Proofs are Narrow – Resolution made Simple

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Abstract

The *width* of a Resolution proof is defined to be the maximal number of literals in any clause of the proof. In this paper we relate proof width to proof length (=size), in both general Resolution, and its tree-like variant. The following consequences of these relations reveal width as a crucial "resource" of Resolution proofs.

In one direction, the relations allow us to give *simple, unified* proofs for almost all known exponential lower bounds on size of resolution proofs, as well as several interesting new ones. They all follow from width lower bounds, and we show how these follow from natural expansion property of clauses of the input tautology.

In the other direction, the width-size relations naturally suggest a simple dynamic programming procedure for automated theorem proving - one which simply searches for small width proofs. This relation guarantees that the running time (and thus the size of the produced proof) is at most quasi-polynomial in the smallest tree-like proof. This algorithm is never much worse than any of the recursive automated provers (such as DLL) used in practice. In contrast, we present a family of tautologies on which it is exponentially faster.

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