Deterministic Amplification of Space-Bounded Probablilistic Algorithms

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Abstract
This paper initiates the study of deterministic amplification of space-bounded probabilistic algorithms. The straightforward implementations of known amplification methods cannot be used for such algorithms, since they consume too much space. We present a new implementation of the Ajtai-Komlos-Szemeredi method, that enables to amplify an S-space algorithm that uses r random bits and errs with probability ϵ to an $O(kS)$-space algorithm that uses $r+O(k)$ random bits and errs with probability $\epsilon^{\omega(k)}$.

This method can be used to reduce the error probability of BPL algorithms below any constant, with only a constant addition of new random bits. This is weaker than the exponential reduction that can be achieved for BPP algorithms by methods that use only $O(\gamma)$ random bits. However, we prove that any black-box amplification method that uses $O(\gamma)$ random bits makes at most p parallel simulations reduces the error to at most $\epsilon^{O(\gamma)}$. Hence in BPL, where p should be a constant, the error cannot be reduced to less than a constant. This means that our method is optimal with respect to black-box amplification methods, but use $O(\gamma)$ random bits.

The new implementation of the AKS method is based on explicit constructions of constant-space online extractors and online expanders. These are extractors and expanders, for which neighborhoods can be computed in a constant space by a Turing machine with a one-way input tape.