Euclidean sections of ℓ_1^N with sublinear randomness and error-correction over the reals

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Abstract. It is well-known that $\mathbb{R}^N$ has subspaces of dimension proportional to N on which the ℓ_1 and ℓ_2 norms are uniformly equivalent, but it is unknown how to construct them explicitly. We show that, for any $\delta > 0$, such a subspace can be generated using only N^δ random bits. This improves over previous constructions of Artstein-Avidan and Milman, and of Lovett and Sodin, which require $O(N \log N)$, and $O(N)$ random bits, respectively.

Such subspaces are known to also yield error-correcting codes over the reals and compressed sensing matrices. Our subspaces are defined by the kernel of a relatively sparse matrix (with at most N^δ non-zero entries per row), and thus enable compressed sensing in near-linear $O(N^{1+\delta})$ time.

As in the work of Guruswami, Lee, and Razborov, our construction is the continuous analog of a Tanner code, and makes use of expander graphs to impose a collection of local linear constraints on vectors in the subspace. Our analysis is able to achieve *uniform* equivalence of the ℓ_1 and ℓ_2 norms (independent of the dimension). It has parallels to iterative decoding of Tanner codes, and leads to an analogous near-linear time algorithm for error-correction over reals.