The Quantum Communication Complexity of Sampling

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Abstract

Sampling is an important primitive in probabilistic and quantum algorithms. In the spirit of communication complexity, given a function $f: X \times Y \rightarrow \{0,1\}$ and a probability distribution D over $X \times Y$, we define the sampling complexity of (f,D) as the minimum number of bits Alice and Bob must communicate for Alice to pick $x \in X$ and Bob to pick $y \in Y$ as well as a value z such that the resulting distribution of (x,y,z) is close to the distribution $(D,f(D))$.

In this paper we initiate the study of sampling complexity, in both the classical and quantum model. We give several variants of the definition. We completely characterize some of these variants, and give upper and lower bounds on others. In particular, this allows us to establish an exponential gap between quantum and classical sampling complexity for the set disjointness function.