For China

What is most important for math in the near future?

1. Computerized library of math knowledge - computerized version of Bourbaki
2. Connecting pure and applied mathematics

Tell about Bourbaki? Why they did not succeed. Gradual process from hyperlinked math text to computer verifiable text.

20-th century greatest advances in algebra, number theory, topology - compare Fields medals. Most of these advances come from combining visul intuition with algebraic/symbolic methods. Much less in analysis. These advances had very little impact on applications of mathematics. Exceptions: error correcting codes, basic cryptography, physics (sort of...).

We discovered very fundamental classes of objects (e.g categories, sheaves, cohomology, simplicial sets). May be as fundamental as groups .... but we do not use them to solve problems outside math.

One of the reasons sociological. Few people have knowledge both of modern math and of a field outside math where applications are possible. To notice that the methods mentioned above are applicable one usually needs first to generalize the problem abstracting oneself from the intuition which comes attached to the problem. Example ? This is hard to teach and this way of looking at things is contrary to the approach which is very popular in the US especially in biology where the value of new knowledge is estimated by how much closer to some practical goal it makes us feel. To get money for research one often has to convince people around that this research is going to enable us to solve some practical problem. In many cases there is a big difference between how easy it is to convince people that something is useful and how useful it turns out to be at the end.

Stock market. Biotech. "Stories". "The billion dollar drug" - a convincing story is often just that and real breakthroughs might be very hard to sell. Old problem. Has no effective solution. The only solution known is to support basic research i.e. essentially encourage people to spend money to satisfy their curiosity and their sense of beauty in science - very inefficient but has no alternative.

To apply mathematics to a problem effectively the first move is exactly the opposite. Instead of trying to concentrate on future applications to the real life to abstract yourself from real life and to look at the problem as at a formal game/puzzle. Because of that new math especially at first often looks like a move away from the real problem.

General hypocrisy of the system which is perpetuated by the push to explain the "usefulness" of the proposed research. Connect to teaching calculus. Another exaple of hypocrisy. How to build an efficient system? State support + no politics + competition. Recognition that pumping money in a field does not lead to more advances after an initial part of the curve - instead leads to more hype and less serious research.

Need to see where we are now, how we got there and where to go next. Most of what I will say is not very unexpected at least to a western mathematician. It would be interesting to compare this view with the view of a Chinese mathematicians and scientists.

I would like to distinguish two different meaning of the phrase "modern mathematics". We may either use it to speak about mathematics as a field of knowledge or as an institution. In the first sense mathematics is a collection of theories, conjectures, theorems etc. which are contained in texts. Most of these texts are mathematical. Others which provide the connection between mathematics and other branches of learning are not.

In the second sense mathematics it is a collection of professional societies, mathematics departments, research institutes etc. which ultimately consist of people. Most of these people are mathematicians. Others which provide the connection between mathematics and other parts of society are not. In what follows I will speak of mathematics in the first sense. To hear my views of mathematics in the second sense please come to my lecture at Wuhan University.

When I talk about some area of mathematics (in the first sense) being more or less active during a certain period of time I mean activity in innovation. That is how much new ideas and results appeared in this area during this period. For example, I do not consider calculus to be an active part of mathematics today even though the number of texts in calculus which appear every year is quite high.

Where we are. The parts of mathematics which seem to be most active in innovation today are algebraic geometry, low dimensional topology and geometry, and theoretical computer science and combinatorics. Of these three areas he first one is the most mature one and the third one is the most recent. If we include areas which are slightly less active but still very important we will get a picture which might be described by saying that today most activity in mathematics occurs in the areas of geometry, number theory and algebra on the one hand and combinatorics and theoretical computer science on the other.

How we got there. Mathematics developed rapidly for a considerable time. It is difficult to put an exact date on the beginning of the present expansion. Let us say that it lasts for about 200 years. If we try to visualize the development of mathematics as a kind of branching process we my ask ourselves about the "parametrs" of this process.

The particular type of mathematics which is prevalent today developed very rapidly for about 50 years - from 1950 to 2000. Now it might be slowing down.

When we look at the situation in mathematics today we see both things which are common to any free activity which was innovating rapidly for a sufficiently long time and things particular to contemporary mathematics.

Among common things:

1. Unbalanced development - the branches which grow rapidly take resources from the less successful branches. The grows of the less successful branches slow down or stops entirely.

In mathematics this is expressed by the existence of fashionable and unfashionable fields. Some of the unfashionable fields which will probably turn out to be important in the future essentially died out. It is obviously hard to give a contemporary example - if I knew what will be hot in the future and is neglected today I would make money on the stock market!

2. Loss of unity - new directions appear frequently and there is not enough time to establish connections between them. The same kind of argument may be discovered several times independently in several subfield without people being aware about it. In mathematics this is expressed by the fact that there is hardly aneone today who would have understood the connections between different branches in