

## WAM IAS Program - Graph Theory Exercises (Lecture 4)

### Exercise 1:

Prove that  $R(3, 4) = 9$ .

### Exercise 2:

Let  $P_4$  denote the path with 4 edges. By considering  $P_4$  and  $K_{1,3}$ , prove that  $r(H_1, H_2) \geq \min\{r(H_i, H_i) : i = 1, 2\}$  is not necessarily true.

### Exercise 3:

Let  $m \geq 2$  and  $n \geq 2$  be such that  $m - 1$  divides  $n - 1$ . Prove that for any tree  $T_m$  with  $m$  vertices,  $r(T_m, K_{1,n}) = n + m - 1$ .

### Exercise 4:

Prove that for every  $m \geq 1$  there exists  $n$  such that the following holds. For every  $n \times n$  matrix with entries in  $\{0, 1\}$ , there exists a principal  $m \times m$  submatrix (i.e. one whose diagonal entries are on the main diagonal) such that all entries below the diagonal are the same, and all entries above the diagonal are the same.

### Exercise 5:

Prove that  $2^k \leq R_k(3, \dots, 3) - 1 \leq 1 + k(R_{k-1}(3, \dots, 3) - 1)$ .

### Exercise 6:

Prove that if the set  $\{1, 2, \dots, 2 + \lfloor ek! \rfloor\}$  is partitioned into  $k$  classes then the equation  $x + y = z$  is solvable in at least one of the classes (that is, for some class  $C$  there exist  $x, y, z \in C$  satisfying  $x + y = z$ ).