GRASSMANNIANS AND PLÜCKER EMBEDDING

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There are far too many problems to do here in an hour, or maybe even a day. You should choose the problems that look interesting to you that you might have an idea about how to start.

(1) Draw a picture of $V(x^2 - y)$.
(2) Find copies of $\mathbb{C}^3$, $\mathbb{C}^2$ and $\mathbb{C}^1$ in $\mathbb{P}^4$. Can you make it so that you find $\mathbb{C}^2$ that is not a subset of the copy of $\mathbb{C}^3$ you found and the copy of $\mathbb{C}^1$ that you found is not a subset of either the $\mathbb{C}^2$ or $\mathbb{C}^3$ you found?
(3) Describe explicitly, that is list all of the points in, the set $V(x_0 x_1) \subseteq \mathbb{P}^1$.
(4) Find the Plücker coordinates for $\operatorname{span}\langle e_1 + 2e + 2, e_3 + 3_4\rangle \subseteq \mathbb{C}^4$.
(5) Find the subspace of $\mathbb{C}^4$ with Plücker coordinates $p_{12} = 1, _{13} = 2, p_{14} = 1, p_{23} = 1, p_{24} = 2$ and $p_{34} = 3$.
(6) Find the embedding of $\mathrm{Gr}(2,4)$ into projective space. What equations does the image of this embedding satisfy?
(7) Find a sub variety of a projective space (say which one!) that is isomorphic to $\mathrm{Gr}(3,5)$.
(8) Write down the Plücker relations for the embedding of $\mathrm{Gr}(3,5)$ into $\mathbb{P}^{\binom{5}{3}-1}$.
(9) Show that $\mathrm{Gr}(1,n)$ is isomorphic to $\mathrm{Gr}(n-1,n)$.
(10) Show that the list of maximal minors (up to scale) of a matrix A identifies its row span completely. That is, show that if the row span of A and B is the same if and only if for every set of columns σ $\det A_\sigma = \lambda \det B_\sigma$, for some $\lambda \in \mathbb{C} - \{0\}$. Here M_σ is the determinant of the sub matrix of M with all rows and columns σ.
(11) Prove that (up to sign) there is only one Plücker relation in $\mathrm{Gr}(2,4)$.
(12) Show that the Plücker relations hold on the image of the embedding $\mathrm{Gr}(k,n) \hookrightarrow \mathbb{P}^{\binom{n}{k}-1}$.
(13) Prove that the Zariski topology is a topology.
(14) Why are the Zariski closed sets in $\mathbb{P}^n$ given by the zeros of homogeneous polynomials?