Description:

Symplectic geometry is a geometry of even dimensional spaces in which area measurements, rather than length measurements, are the fundamental quantities. This course will introduce symplectic manifolds, starting with symplectic vector spaces and examples in $\mathbb{R}^{2n}$. It will culminate with the foundational non-squeezing theorem that reveals the ever-present tension between the rigid geometric and the soft topological features of symplectic manifolds.

Background:

Multivariable calculus and linear algebra are both prerequisites for this course. Advanced calculus or an exposure to some topology would be helpful but is not required.

Recommended Texts:

For something very concise: Spivak's *Analysis on Manifolds*, especially Ch. 4 and 5. For something detailed: Lee's *Introduction to Smooth Manifolds*, especially Ch. 1, 3-5 and 8-9.

These texts can help you understand manifolds and differential forms. No prior knowledge of manifolds or differential forms is required and any explicit use of differential forms will be relegated to select exercises – for those who are familiar with forms or would like to begin working with them.

Lecture 1: Symplectic Matrices

In this lecture I will motivate the definition of a symplectic matrix (in the context of classical mechanics), introduce symplectic vector spaces, draw comparisons with the more familiar case of a vector space equipped with an inner product, and explain the sense in which signed area is the fundamental quantity one can measure in a symplectic vector space.

Lecture 2: Of Planes and Parallelepipeds

This lecture will be devoted to understanding the different types of subspaces of a symplectic vector space (especially symplectic and Lagrangian subspaces) and what, beyond signed area, one can measure in a symplectic vector space.

Lecture 3: Submanifolds of $(\mathbb{R}^{2n}, \omega_0)$ – Symplectic, or Not

In this lecture we will consider examples of submanifolds (the non-linear analog of subspaces) of $\mathbb{R}^{2n}$ equipped with the standard symplectic structure ω_0, emphasizing symplectic and Lagrangian submanifolds.

Time permitting we will prove the following:
Theorem: There are no closed symplectic surfaces in $(\mathbb{R}^{2n}, \omega_0)$.
Theorem: The symplectic structure ω_0 on $\mathbb{R}^4$ induces a contact structure on S^3.

Lecture 4: Constructing Symplectic Manifolds

In this lecture we will try to build some manifolds out of pieces of $(\mathbb{R}^2, \omega_0)$ and $(\mathbb{R}^4, \omega_0)$. Our efforts will lead us to prove the first and motivate the second of these two theorems:
Theorem: Any orientable surface is symplectic.
Theorem: For any $n > 2$, the sphere S^{2n} admits no symplectic structure.

Lecture 5: The Non-squeezing Theorem

We will prove the affine non-squeezing theorem and consider the implications of the general non-squeezing theorem and the symplectic camel theorem.