1 Homework I

1. Eigenvectors and eigenvalues

Let

\[
K = \begin{pmatrix}
.75 & .25 \\
.25 & .75 \\
\end{pmatrix}.
\]

- Show that 1 and 1/2 are eigenvalues of \( K \) and find the eigenvectors. Express \( K \) as \( PDP^{-1} \) where \( D \) is diagonal and \( P \) is orthonormal.
- Let \( K' = \alpha K \) for real \( \alpha \neq 0 \). Find the eigenvalues and eigenvectors of \( K' \).
- Consider the \( m \)th power of \( K \), \( K^m \), for \( m > 0 \). Find the eigenvectors and eigenvalues of \( K^m \).

2. Gaussian random variables

Recall that a Gaussian random variable \( x \) with mean \( \mu \) and standard deviation \( \sigma \) has density:

\[
\frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.
\]

We will use the notation \( x \sim \mathcal{N}(\mu, \sigma) \).

- Suppose \( a \) and \( b \) are constants. Show that if \( x \sim \mathcal{N}(0, 1) \), then \( bx + a \sim \mathcal{N}(a, b) \).
- Show that if \( x_1 \sim \mathcal{N}(\mu_1, \sigma_1) \) and \( x_2 \sim \mathcal{N}(\mu_2, \sigma_2) \), then \( x_1 + x_2 \sim \mathcal{N}(\mu_1 + \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2}) \).
- Suppose \( x \sim \mathcal{N}(0, \sigma) \). What is \( \mathbb{E}(x^2) \)? What is the distribution of \( x^2 \)?