Metaplectic Ramanujan Conjecture and Ternary Quadratic Forms Over Function Fields

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Abstract
The Ramanujan conjecture states that for a holomorphic cusp form \( f(z) = \sum_{n \in \mathbb{N}} \lambda_f(n) e(nz) \) of weight \( k \), the coefficients \( \lambda_f(n) \) satisfy the bound \( |\lambda_f(n)| \ll n^{(k-1)/2+\epsilon} \). In the case where \( k \) is an integer this is a celebrated theorem of Deligne which he proved by reducing to a case of the Weil conjectures. In the case of half-integral weight the conjecture remains wide open, though non-trivial bounds towards it have been established by Iwaniec and Duke. We will focus on the function field case \( F_p(T) \), where we formulate and prove the analogue of the half-integral Ramanujan conjecture. Our proof makes use of Drinfeld’s results relating cusp forms on \( GL(2)/F_p(T) \) to galois representations, as well as developing the Shimura correspondence and a Waldspurger formula in this function setting. As our main application, we give a solution (which is moreover effective) for representing elements in \( F_p[T] \) by a given ternary quadratic form with coefficients in this ring. This is joint work with Ali Altug.