Ramsey Theory for Metric Spaces

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Abstract

Ultrametrics are special metrics satisfying a strong form of the triangle inequality: For every x, y, z, $d(x,z) \Leftarrow max\{d(x,y), d(y,z)\}$. We consider Ramsey-type problems for metric spaces of the following flavor:

Every metric space contains a "large" subset having approximate ultrametric structure.

The following theorem implies a variety of Ramsey-type theorems for compact metric spaces with different notions of "size":

> *For every $e > 0$, every compact metric space X and every probability measure μ on X, there exists a subset S of X and a probability measure ν supported on S such that S is an approximate ultrametric upto distortion $9/e$, and for every ball $B(x,r)$ in X, $\nu(B(x,r)) \Leftarrow \mu(B(x,Cr))^{1-e}$, where $C = C_e > 1$ depends only on e.*

Those Ramsey-type theorems, besides their intrinsic interest, have applications for algorithms (approximate distance oracles, lower bounds for online problems), metric analysis (Lipschitz surjections onto the n-dimensional cube), and probability (Talagrand's majorizing measure theorem).